

**INDIAN SCHOOL SOHAR  
FIRST TERM EXAM 2013  
MATHEMATICS**

**Class – XI**

**Date -19/09/13**

**Time ; 3 Hrs**

**Marks ; 100**

**General Instructions**

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1. All questions are compulsory.
  2. The question paper consist of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.
  3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
  4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
  5. Use of calculators is not permitted.
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**SECTION - A**

1. Let  $A = \{1, 2\}$  and  $B = \{3, 4\}$ . Find the number of relations from A to B.
2. Find the radian measure of  $25^\circ$
3. What is the amplitude of the product of the complex numbers  
 $(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$  and  $(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6})$
4. Find the number of permutations of the letters of the word ALLAHABAD.
5. Write each of the following statements in the form “if-then”  
“To get an A+ in the class, it is necessary that you do all the exercises of the book.”
6. State whether the “Or” used in the following statements is “exclusive “or” inclusive. Give reasons for your answer.  
“To apply for a driving license, you should have a ration card or a passport.”

**SECTION – B**

7. Using the properties of sets prove that  $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$  for any three sets A, B and C
8. Find the domain and range of the function defined by  $f(x) = \frac{x^2}{x^2 + 1}$
9. Find the general solution of the equation  $\cos 3x + \cos x - \cos 2x = 0$

**[ OR ]**

Prove that  $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$

10. By using principle of mathematical induction prove that

$$\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{2n+1}{n^2}\right) = (n+1)^2, n \in N$$

11. By using principle of mathematical induction prove that  $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$  is divisible by 25

**[ OR ]**

For every positive integer  $n$ , prove that  $7^n - 3^n$  is divisible by 4.

12. A person is represented by a complex number  $z = x + iy$ . If a person is represented only by  $x$  then he is not sensitive towards environment and if a person is represented only by 'y' then he is sensitive towards environment. If a person is related by the relation  $\left|\frac{z-5i}{z+5i}\right| = 1$ , do you think the person is eco friendly?

13. Find the modulus and argument of  $\frac{i-1}{\sin \frac{\pi}{6} + i \cos \frac{\pi}{6}}$

14. Sixteen men complete with one another in running, swimming, and riding. How many prize list could be made if there were altogether 6 prizes of different values one for running, 2 for swimming and 3 for riding? Write the importance of sports in your day to day life.

15. Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50<sup>th</sup> word?

16. Show that the middle term in the expansion of  $(1+x)^{2n}$  is  $\frac{1.3.5.7 \dots (2n-1)}{n!} 2^n x^n$  where  $n$  is a positive integer.

17. Prove that  $\sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} = 2$

[ OR ]

Prove that  $\frac{2 \cos 2A + 1}{2 \cos 2A - 1} = \tan(60 + A) \tan(60 - A)$

18. A manufacturer has 600 litres of a 12 % solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15 % but less than 18 %.

19. Find the sum of all possible products of the first  $n$  natural numbers taken two at a time.

[OR ]

Find the sum to  $n$  terms of the series:  $5 + 11 + 19 + 29 + 41 \dots$

### SECTION – C

20. Find the sum to  $n$  terms of the sequence  $\left(x + \frac{1}{x}\right)^2, \left(x^2 + \frac{1}{x^2}\right)^2, \left(x^3 + \frac{1}{x^3}\right)^2, \dots$

21. In a pollution study of 1500 Indian rivers the following data were reported .520 were polluted by sulphur compounds, 335 polluted by phosphate, 425 polluted by crude oil, 100 were polluted by both crude oil and sulphur compounds, 180 were polluted by both sulphur compounds and phosphate, 150 were polluted by both phosphates and crude oil. How many rivers were polluted by exactly one of these impurities if 878 rivers were declared as polluted rivers. What is the importance of keeping the rivers without pollution? Write any two suggestions to keep the rivers clean.

22. Show that  $\tan(x-y) + \tan(y-z) + \tan(z-x) = \tan(x-y) \tan(y-z) \tan(z-x)$

23. Find  $a$ ,  $b$  and  $n$  in the expansion of  $(a + b)^n$  if the first three terms of the expansion are 729, 7290 and 30375, respectively.

24. Exhibit graphically the solution sets of the following linear in equations

$$2x + 3y \geq 6, \quad x - 2y \leq 2, \quad 3x + 2y \leq 12, \quad -3x + 2y \leq 3, \quad x \geq 0, \quad y \geq 0$$

What are the values of life you have learned while shading the common region of all these inequalities?

25. If in any triangle ABC  $\frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15}$ , then prove that  $\frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$

[OR]

In any triangle ABC prove that  $\frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - b^2}{a^2} \sin 2B + \frac{a^2 - b^2}{c^2} \sin 2C = 0$

26. Between 1 and 31,  $m$  numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7<sup>th</sup> and  $(m - 1)$ <sup>th</sup> numbers is 5 : 9. Find the value of  $m$ .

[OR]

Find the sum of the following series to 'n' terms  $\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \dots$

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