INDIAN SCHOOL SOHAR
FIRST TERM EXAM 2014
MATHEMATICS

Class – XII **Time ; 3 Hrs**
Date -22/09/14 **Marks ; 100**

General Instructions

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1. All questions are compulsory.
2. The question paper consist of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION – A

1. Let f : R → R be defined by f (x) = x^2 + 1. Then, what are the pre-images of 17 and – 3?
2. Evaluate $\tan^{-1}\sin\left(\frac{-\pi}{2}\right)$
3. Show that the points (1, 0), (6, 0), (0, 0) are collinear.
4. If y = secx + tanx , write the simplest form of $\frac{dy}{dx}$
5. What is the maximum and minimum value of the function $f(x) = -|x+5| + 7$
6. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} . \vec{b}$ = 8 and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

SECTION – B

7 .Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$

[OR]

Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

8 . Solve the following for ‘x’ $\cos^{-1}\frac{x^2-1}{x^2+1}+\tan^{-1}\frac{2x}{x^2-1}=\frac{2\pi}{3}$

9 . Let Z be the set of all integers and R be the relation on Z defined as R = {(a, b) ; a, b ε Z, and (a - b) is divisible by 5.} Prove that R is an equivalence relation.

10 . Consider f : R+ →[4, ∞] given by f(x) = x^2+4 . Show that f is invertible with the inverse (f^{-1})of f given by f^{-1}(y) = $\sqrt{y-4}$, where R+ is the set of all non-negative real numbers.

11 . Using the properties of determinants prove that $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz + xy + yz + zx$

12 . Using the properties of determinants prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

13 . If x sin (a + y) + sin a cos (a + y) = 0 , show that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

[OR]

Find the value of ‘k’ for which the function f defined as

$$f(x)=\begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2}, & x\neq 2 \\ k, & x=2 \end{cases}$$

is continuous at x = 2

14 . If y = e^x (sin x + cos x), then show that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

[OR]

If x =a (cost + t sin t) and y = a (sin t – t cos t) then find $\frac{d^2y}{dx^2}$

15 . Find the intervals in which the function f given by f(x) = sin x - cos x, 0 ≤ x ≤ 2π is strictly increasing or strictly decreasing. Write any two values learned from this problem.

16 . Check whether the following four points with the position vectors $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$ and $2\hat{i} + 5\hat{j} + 10\hat{k}$ are co planar .

17 . The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

18 . Water is dripping out at a steady rate of 1 cu cm/sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4 cm, find the rate of decrease of slant height, where the vertical angle of the conical vessel is $\frac{\pi}{6}$

[OR]

Using differentials, find the approximate value of f(2 . 01), where $f(x) = 4x^3 + 5x^2 + 2$.

19 . Find the shortest distance between the following pair of lines and hence write whether the lines are intersecting or not : $\frac{x-1}{2} = \frac{y+1}{3} = z$; $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$

SECTION – C

20 . A firm manufactures jute bags and cloth bags. The total number of items it can manufacture is at most 24. A Jute bag requires 1 hour to be made while a cloth bag requires only half an hour. The maximum number of hours available per day is 16 hours. If the profit on a jute bag is Rs.30 and on a cloth bag is Rs.20, how many bags of each type must be made for maximum profit? Solve it graphically. The manufacturer wants to replace cloth bags by plastic bags to increase his profit margin. Is it a good idea? If not, give reasons.

21 . Show that the equation of normal at any point on the curve $x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$ is $4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$.

[OR]

If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

22 . If the sum of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$

[OR]

A manufacturer can sell x items at a price of Rs. $\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to earn maximum profit.

23 . Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$; $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing the lines.

24 . Find the distance of the point (- 2, 3, - 4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane 4x + 12y - 3z + 1 = 0.

25 . Differentiate the following function with respect to x $f(x)=\tan^{-1}\left(\frac{1-x}{1+x}\right)-\tan^{-1}\left(\frac{x+2}{1-2x}\right)$

[OR]

Find the value of ‘a’ for which the function f defined as $f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), & x\le 0 \\ \frac{\tan x-\sin x}{x^3}, & x>0 \end{cases}$

is continuous at x = 0

26 . The cost of 4 chocolates, 3 samosas and 2 apples is Rs.60 and that of 2 chocolates, 4 samosas, and 6 apples is Rs.90. The cost of 6 chocolates, 2 samosas, 3 apples is Rs.70. Find the cost of each item by matrix method. What do you think is the healthiest diet? Suggest an item that could replace chocolates and samosas to make the diet healthier?

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