

INDIAN SCHOOL SOHAR
UNIT TEST 2014 – 2015
MATHEMATICS

Class: XII
Date: 27/05/14

Time: 2 Hrs
Marks: 50

General Instructions

1. All questions are compulsory.
2. The question paper consist of 15 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 5 questions of four marks each and section C comprises of 4 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 01 question of four marks each and 01 question of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION - A

1. Find the number of all onto functions from the set $\{1, 2, 3, \dots, n\}$ to itself.
2. Write the domain and range of $\sec^{-1}(x)$
3. Find the value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
4. Find the direction cosines of a vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$
5. Show that the points A (2, 3, -4), B (1, -2, 3) and C (3, 8, -11) are collinear.
6. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.

SECTION – B

7. Find the length of the foot of the perpendicular drawn from the point (2,-1,5) to the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$

[OR]

Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k} \quad \text{and} \quad \vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

8. Let $A = N \times N$ and $*$ be a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that $*$ is commutative and associative. Also, find the identity element for $*$ on A, if any.

9. Solve for 'x' ' If $\sin^{-1}(1-x) + 2 \sin^{-1} x = \frac{\pi}{2}$

10. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j},$ and \hat{k} $\vec{\alpha} = 3\hat{i} - \hat{j}, \vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$

11. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

Distance in (km)		
From / To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, formulate the above LPP and obtain the objective function of the problem

SECTION – C

12. Consider $f : R \rightarrow (-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is injective, surjective and find the inverse of the function, ensure the inverse using the composition of functions.

13. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1500 is made on each executive class ticket and a profit of Rs 1000 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

14. Find the equation of the line passing through the point (4, 6, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$

[OR]

Find the equation of the plane containing the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-6}{7}$

15. Show that if the vectors \vec{a}, \vec{b} and \vec{c} are co planar ,then $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{a} + \vec{c}$ are co planar