INDIAN SCHOOL SOHAR
SECOND TERM EXAM 2014
MATHEMATICS

Class – XII **Time ; 3 Hrs**
Date -27/11/14 **Marks ; 100**

General Instructions

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1. All questions are compulsory.
2. The question paper consist of 26 questions divided into three sections A, B and C. Section A comprises of 06 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION – A

1. For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other?
2. If f(x) is an invertible function, find the inverse of f(x) = $\frac{3x^2 - 2}{5}$.
3. Evaluate $\int \frac{x + \cos 6x}{3x^2 + \sin 6x}\, dx$
4. If $\int \left(e^{ax} + bx\right) dx = 4e^{4x} + \frac{3x^2}{2}$ find the values of 'a' and 'b'
5. If the equation of the line AB is $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$ find the direction cosines of the line which is parallel to AB
6. If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$ find the value of 'x'

SECTION – B

7. Define a binary operation $*$ on the set {0, 1, 2, 3, 4, 5} as $a * b = \begin{cases} a+b & \text{if } a+b < 6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$

   Show that zero is the identity for this operation and each element 'a' of the set is invertible with 6 – a being the inverse of a.

8. If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$, then prove that $9x^2-12xy\cos\theta+4y^2=36\sin^2\theta$

9. Using the properties of determinants prove that $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}=4a^2b^2c^2$

10. Using the properties of determinants solve the following for x $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix}=0$

11. If $y=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$,then prove that $\frac{dy}{dx}=\sqrt{a^2-x^2}$

12. Find the value of k so that the function f defined by $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x}, & if\ x\neq\frac{\pi}{2} \\ 3 & if\ x=\frac{\pi}{2} \end{cases}$

is continuous at $x=\frac{\pi}{2}$

13. Evaluate $\int\frac{(5x+3)}{\sqrt{x^2+4x+10}}\,dx$

[OR]

Evaluate $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

14. Evaluate $\int_{-1}^{3/2}\left|x\sin\pi x\right|\,dx$

[OR]

Evaluate $\int_{0}^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}\,dx$

15. Evaluate $\int\frac{dx}{\cos(x-a)\cos(x-b)}$

[OR]

Evaluate $\int\frac{(x-3)}{(x-1)^3}\quad e^x\ \ dx$

16. Form the differential equation representing the family of ellipses having foci on x-axis and centre at the origin.

[OR]

Form a differential equation representing the given family of curves by eliminating arbitrary constants a and b. $y = a\,e^{3x} + b\,e^{-2x}$

17. Find the shortest distance between the lines $\vec{r} = (1+2\lambda)\hat{i} + (1-\lambda)\hat{j} + \lambda\hat{k}$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

18. Find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

19. A and B throw a pair of die turn by turn. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of A getting the prize is 9/17.Suggest some ways to increase the probability of reaching success.

SECTION – C

20. The management committee of a school decided to award some of its students for honesty, some for helping others and some other for organizing the awareness program keep the school neat and clean. The sum of all the awardees is 24. One more than the sum of the awardees for helping others and organizers of awareness programmes is four times of the awardees for honesty. Three times the awardees for helping others is four less than the sum of the other awardees. Find the number of awardees in each category .If the management wish to increase the number of awardees in future, which value you will suggest and why?

21. A wire of length 32 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

[OR]

For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

22. Find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

[OR]

Using the method of integration, find the area of the region bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

23. Find the equation of the perpendicular drawn from the point (1, –2, 3) to the plane $2x - 3y + 4z + 9 = 0$. Also find the co-ordinates of the foot of the perpendicular.

24. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is not late. What is the probability that he comes by train? Write the importance of punctuality.

25. Find the particular solution of the differential equation $(\tan^{-1}y - x) \, dy = (1 + y^2) \, dx$, given that when x = 0, y = 0

[OR]

Find the particular solution of the differential equation $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x$, given that when x = 0, y = 1

26. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as a linear programming problem and solve graphically.