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INDIAN SCHOOL SOHAR
FIRST TERM EXAM 2015
MATHEMATICS

Class: XII | **Time: 3 Hrs**
Date: 20/09/15 | **Marks: 100**

General Instructions

a. *All questions are compulsory.*
b. *The question paper consist of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.*
c. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
d. *There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six mark each. You have to attempt only one of the alternatives in all such questions.*
e. *Use of calculators is not permitted.*

SECTION – A

1. Find $f \circ g(x)$ if $f(x) = |x|$ and $g(x) = |5x - 2|$

2. Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

3. What is the value of the determinant $\begin{vmatrix} 4 & a & b+c \\ 4 & b & a+c \\ 4 & c & a+b \end{vmatrix}$?

4. If $y = \log x$ find Δy when $\Delta x = 0.003$ and x = 3

5. Find the value of ‘ p’ if $\left(2\hat{i} + 6\hat{j} - 27\hat{k}\right) \times \left(\hat{i} + 3\hat{j} + p\hat{k}\right) = \vec{0}$

6. Evaluate $\int \frac{(1+\log x)^2}{x} dx$

SECTION – B

7. Evaluate $\int \frac{dx}{1+\cot x}$ **[OR]** Evaluate $\int \frac{dx}{\cos(x-a)\cos(x-b)} dx$

8. Evaluate $\int \frac{x+2}{\sqrt{x^2+2x+3}} dx$

9. Show that $2\tan^{-1}\left\{\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right\} = \tan^{-1}\frac{\sin\alpha \cdot \cos\beta}{\cos\alpha + \sin\beta}$

10. Solve the equation $\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

11. Let f: N →R be a function defined as f (x) = $4x^2 + 12x + 15$. Show that f: N→ S, where, S is the range of f, is invertible. Find the inverse of f.

12. Express the following matrix as the sum of the symmetric and skew symmetric matrix

$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

[OR]

If A = $\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, Show that I+A= I-A$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

13. Prove the following using properties of determinants $\begin{vmatrix} a+bx^2 & c+dx^2 & p+qx^2 \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix} = (x^4-1)\begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}$

14. Using properties determinants, prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

15. If the function 'f' as defined below is continuous at x = 0, find the values of 'a','b', and 'c'

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & , x<0 \\ c & , x=0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{b\,x^{3/2}} & , x>0 \end{cases}$$

16. If $y = \log\left[x + \sqrt{x^2 + a^2}\right]$ show that $(x^2 + a^2)\dfrac{d^2y}{dx^2} + x.\dfrac{dy}{dx} = 0$

[OR]

If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$ show that $(1-x^2)\dfrac{d^2y}{dx^2} - x.\dfrac{dy}{dx} - a^2y = 0$

17. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 5$ $|\vec{b}| = 12$ and $|\vec{c}| = 13$ and $\vec{a} + \vec{b} + \vec{c} = 0$. Find the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$

[OR]

If $\vec{a}, \vec{b}$ are unit vectors such that the vectors $\vec{a} + 3\vec{b}$ is perpendicular to $7\vec{a} - 5\vec{b}$ and $\vec{a} - 4\vec{b}$ is perpendicular to $7\vec{a} - 2\vec{b}$ then find the angle between $\vec{a}$ and $\vec{b}$

18. Find the shortest distance between the lines $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$

19. The amount of pollution content added in air in a city due to 'x' diesel vehicles given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

SECTION – C

20. A village has 500 hectares of land to grow two types of plants, X and Y .The contribution of total amount of oxygen produced by plant X and plant Y are 60% and 40% per hectare respectively. To control weeds, a liquid herbicide has to be used for X and Y at rates of 20 litres and 10 liters per hectare respectively. Further no more than 8000 liters of herbicides should be used in order to protect aquatic animals in a pond which collects drainage from this land .How much land should be allocated to each crop so as to maximize the total production of oxygen ?

How do you think excess use of herbicides affects our environment?
What are the general implications of this question towards planting tree around us?

21. Show that the area of the triangle formed by the tangent and the normal at the point (a, a) on the curve $y^2 (2a - x) = x^3$ and the line $x = 2a$ is $\frac{5a^2}{4}$ sq. units.

[OR]

Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12cm is 16cm.

22. Evaluate $\int \frac{x^2}{x^4 + 1} dx$ **[OR]** Evaluate $\int \frac{6x + 7}{\sqrt{(x-5)(x-4)}} dx$

23. Find the vector equation of the plane passing through the points (2, 1, -1) and (-1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$. Also show that the plane thus obtained contains the line $\vec{r} = (-\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 5\hat{k})$

[OR]

Find the co ordinate of the point where the line through (3, -4, -5) and (2, -3, 1) crosses the plane determined by the points A (1 ,2 , 3) , B (2 , 2 , 1) and C(-1 , 3 , 6) .

24. Let X be a non empty set and *be a binary operation on P(X) (the power set of X) defined by $A * B = A \cup B$ for all $A, B \in P(X)$.Prove that '*' both commutative and associative on P(X) . Find the identity element with respect to '*' on P(X).Also show that P(X) is the only invertible element of P(X).

25. Using matrices solve the following system of linear equations

$x - y + z = 4$; $2x + y - 3z = 0$; $x + y - z = 2$

26. If $y = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ show that $\frac{dy}{dx} = \sqrt{a^2 - x^2}$

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