Number of printed pages – 4

INDIAN SCHOOL SOHAR
SECOND TERM EXAM 2015
MATHEMATICS

Class: XII | **Time: 3 Hrs**
Date: 24/11/15 | **Marks: 100**

General Instructions

a. All questions are compulsory.
b. The question paper consist of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.
c. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

SECTION – A

1. If A = $\lfloor a_{ij} \rfloor$ is a square matrix such that $a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew symmetric matrix.
2. What is the integrating factor of the differential equation $\frac{dy}{dx} - \left(\frac{1}{1+x}\right) y = (1+x)\, e^x$?
3. Write the general solution of the equation $\frac{dy}{dx} = \frac{y}{x}$
4. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with 'y' axis?
5. For what value of 'p', is $\left(\hat{i} + \hat{j} + \hat{k}\right)p$ a unit vector.
6. A line in xy plane makes angle $\frac{\pi}{6}$ with the x axis .Find the direction ratios and direction cosines of the line.

SECTION – B

7. Define a binary operation * on the set A = {0, 1, 2, 3, 4, 5} as
$$a * b = \begin{cases} a+b & if\ \ a+b < 6 \\ a+b-6 & if\ \ a+b \ge 6 \end{cases}$$
Find identity element for this operation and inverse element of 'a'
8. If y = $\cot^{-1}\left(\sqrt{\cos x}\right) - \tan^{-1}\left(\sqrt{\cos x}\right)$, then prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$
9. Prove that $\begin{vmatrix} 2ab & a^2 & b^2 \\ a^2 & b^2 & 2ab \\ b^2 & 2ab & b^2 \end{vmatrix} = -\left(a^3 + b^3\right)^2$

10. Prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

11 .If $f(x) = \begin{cases} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1}, & x \neq 0 \\ k \quad\quad \text{M} & , x = 0 \end{cases}$ *is continuous at* $x = 0$, *find the value of* k

12. Let $y = \tan^{-1}\left[\dfrac{4x}{1+5x^2}\right] + \tan^{-1}\left[\dfrac{2+3x}{3-2x}\right]$ show that $\dfrac{dy}{dx} = \dfrac{5}{1+25x^2}$

[OR]

If $y^{\cos x} + (\tan^{-1} x)^y = 1$, find $\dfrac{dy}{dx}$

13. Evaluate $\int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$ **[OR]** Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{dx}{1+\sqrt{\tan x}}$

14. Evaluate $\int (5x-1)\sqrt{6+5x-2x^2}\, dx$ **[OR]** Evaluate $\int \dfrac{dx}{\sin(x-\alpha)\cos(x-\beta)}$

15. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

[OR]

How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

16.Find the altitude of a parallelepiped determined by the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$ whose base is a parallelogram with the adjacent sides $\vec{a}$ and $\vec{b}$.

[OR]

Find the value of λ if the points A (-1 , 4 , -3) , B (3 , λ ,-5) , C(-3 , 8 , -5) and D (-3 , 2 , 1) are coplanar .

17. Find the co ordinates of the foot of the perpendicular and the perpendicular distance from the point (3, 2, 1) from the plane 2x – y + z + 1 = 0.

18. Form the differential equation representing the family of curves given by $(x - a)^2 + 2y^2 = a^2$, where ‘a ‘is an arbitrary constant.

19 Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$ .Hence find A^{-1}

SECTION – C

20. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred balls were both black.

[OR]

Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ive but 1% are diagnosed as showing HIV+ive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV?

21. Find the area bounded by the lines $x + 2y = 2$, $y - x = 1$, $2x + y = 7$

[OR]

Using integration, find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

22. Consider f : {1, 2, 3} → {a, b, c} and g : {a, b, c} → {apple, ball, cat} defined as f (1) = a, f(2) = b, f(3) = c, g(a) = apple, g(b) = ball and g(c) = cat. Show that f, g and gof are invertible. Find out f^{-1}, g^{-1} and $(gof)^{-1}$ and show that $(gof)^{-1} = f^{-1} o\, g^{-1}$.Explain the term " Literacy " in your view.

23. The lengths of the sides of an isosceles triangle are $9 + x^2$, $9 + x^2$ and $18 - 2x^2$ units. Calculate the area of the triangle in terms of x and find the value of x which makes the area maximum.

[OR]

An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units

24.Find the particular solution of the differential equation $x^2\, dy + y(x + y)\, dx = 0$ given that y = 1 when x = 1

[OR]

Find the particular solution of the following differential equation satisfying the given condition :

$$(3x^2 + y)\frac{dx}{dy} = x \text{ , } x > 0 \text{ } when\ x = 1,\ y = 1$$

25. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

26. A variable plane is at a constant distance p from the origin and meets the co ordinate axes at A , B , C. Show that the locus centroid of the tetrahedron OABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$

****