INDIAN SCHOOL SOHAR
UNIT TEST
MATHEMATICS

Class: XII **Time: 2 Hrs**
Date: 17/05/15 **Marks: 50**

General Instructions

1. All questions are compulsory.
2. The question paper consist of 15 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 5 questions of four marks each and section C comprises of 4 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 01 question of four marks each and 01 question of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION A

1. If $f : R \to R$ be defined by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then find $f \circ f(x)$
2. The binary operation $* : R \times R \to R$ is defined as $a * b = 2a + b$. Find $(2 * 3) * 4$
3. Evaluate $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$
4. Find λ, if $\left(2\hat{i} + 6\hat{j} + 14\hat{k}\right) \times \left(\hat{i} - \lambda\hat{j} + 7\hat{k}\right) = \vec{0}$
5. Vector $\vec{a}$ *and* $\vec{b}$ are such that $\left|\vec{a}\right| = \sqrt{3}$, $\left|\vec{b}\right| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector .Write the angle between $\vec{a}$ *and* $\vec{b}$
6. Find the direction cosines of a line , normal to the plane $7x + y - 2z + 3 = 0$

SECTION – B

7. Solve for ' x ' : $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$

[OR]

Solve for 'x' : $\sin^{-1}\frac{3x}{5} + \sin^{-1}\frac{4x}{5} = \sin^{-1} x$

8. Find the equation of a line through the point (2,1,3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$
9. If R_1 and R_2 equivalence relations. prove that $R_1 \cap R_2$ is also an equivalence relation

P.T.O

10. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$

11. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$,prove that $\vec{a} + \vec{b}$, $\vec{c} + \vec{b}$, and $\vec{a} + \vec{c}$, are coplanar, if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are co planar.

SECTION – C

12. If $f : [1, \infty) \to [2, \infty)$ given by $f(x) = x + \frac{1}{x}$, prove that 'f' is bijective function and find the inverse of 'f'

13. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

14. If a young man rides his motor-cycle at 25 km per hour, he had to spend Rs 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to Rs 5 per km and rate of pollution also increases. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an L.P.P. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?

15. Find the distance of the point (- 2, 3, - 4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane 4x + 12y - 3z + 1 = 0.

[OR]

A variable plane is at constant distance p from the origin and meet the axes in A, B & C. Show that the locus of the centroid of the tetrahedron of a triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$

****