Number of printed pages – 4

INDIAN SCHOOL SOHAR
PRE BOARD EXAM 2015 -2016
MATHEMATICS

Class: XII **Time: 3 Hrs**
Date: 17/01/16 **Marks: 100**

General Instructions

a. All questions are compulsory.
b. The question paper consist of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.
c. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

SECTION – A

1. If A = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ find A^2 hence A^6
2. If y = $e^{3\log x + 2x}$ find $\frac{dy}{dx}$
3. Write the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$
4. Direction cosines of a line are $\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$,find the direction ratios .
5. Find the rate of change of area of a circle with respect to its radius 'r' when r = 3.
6. R is the relation defined on the set of natural numbers N is { (x , y) :2x+y = 41, x,y ε N} Explain why this relation is not transitive with an example . .

SECTION – B

7. Let A be the set of all human beings in a town at a particular time. The relation 'R' is given by $(x, y) \in R \Rightarrow$ x and y are associated with "SAVE THE RARE SPECIES "campaign. Find whether the relation 'R' is reflexive, symmetric, and transitive. How far your education is helpful to the society?
8. Solve the equation $\cos^{-1}\frac{x^2-1}{x^2+1} + \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$

[OR]

Solve the equation $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$

9. If $\Delta_1 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ *and* $\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$ Prove *that* $\Delta_1 + \Delta_2 = 0$

10. Solve the following differential equation $2x^2 \frac{dy}{dx} - 2xy + y^2 = 0$

11. If $f(x) = \begin{cases} 3ax + b & , if \ x > 1 \\ 11, & , if \ x = 1 \\ 5ax - 2b & , if \ x < 1 \end{cases}$ *is continuous at* $x = 1$, *find the value of a and b*

12. Let $y = \cos ec^{-1} x, x > 1$ *prove that* $x(x^2 - 1)\frac{d^2y}{dx^2} + (2x^2 - 1)\frac{dy}{dx} = 0$

[OR]

If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that $\frac{dy}{dx} = \sec x$ *also find* $\frac{d^2y}{dx^2}$ *at* $x = \frac{\pi}{4}$

13. Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$ **[OR]** Evaluate $\int_1^4 (x^2 - x)dx$ *u* sin *g* lim *it of sums*

14. Evaluate $\int \frac{\sin x + \cos x}{\sqrt{9 + 16 \sin 2x}} dx$ **[OR]** Evaluate $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$

15. The probabilities of A, B, C solving problem are $\frac{1}{3}, \frac{2}{7}$ *and* $\frac{3}{8}$ respectively .If all of them try to solve the problem simultaneously ,find the probability that the problem is solved.

[OR]

A and B throw a pair of dice alternately .A wins the game if he gets a total of 6 and B wins if she gets a total of 7. If A starts the game, find the probability winning the game by A in the third throw of pair of dice.

16. If A = $\begin{bmatrix} 3 & 1 \\ 7 & 2 \end{bmatrix}$, find A^{-1} and use this to solve the equation 3x + 7y =4 , x + 2y = 1

17. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect .Also find the point of intersection.

18. Form the differential equation representing the family of ellipses having foci on 'y' axis and centre at the origin.

19 Find the matrix A satisfying the matrix equation $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

SECTION – C

20. A bag contains 4 balls .Two balls are drawn at random and are found to be white .What is the probability that all balls are white .

[OR]

In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.

21. Find the area bounded by the curves $y = 6x - x^2$, and $y = x^2 - 2x$

[OR]

Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ in to three equal parts.

22. Find all points of local maxima and minima and the corresponding maximum and minimum values of the function $f(x) = \sin^4 x + \cos^4 x$, $0 < x < \pi$

23. Manufacturer can sell x items at a price of Rs $\left(5-\frac{x}{100}\right)$ each. The cost of x items is Rs $\left(100+\frac{x}{5}\right)$.Find the number of items he should sell to earn maximum profit.

[OR]

A window has the shape of a rectangle surmounted by an equilateral triangle .If the perimeter of the window is 12 m find the dimensions of the rectangle that will produce the largest area of the window.

24. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1500 is made on each executive class ticket and a profit of Rs 1000 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit? Write the advantages of being simple in our practical life?

25. Find the equation of the planes through the intersection of the planes x + 3y + 6 = 0 and 3x – y – 4z = 0 whose perpendicular distance from the origin is equal to 1.

[OR]

Find the equation of the plane passing through the point (2 , 1 , -1) and (-1 , 3, 4) and perpendicular to the plane x – 2y + 4z = 10 .Also show that the plane thus obtained contains the line $\frac{x+1}{3}=\frac{y-3}{-2}=\frac{z-4}{-5}$.

26. If the vectors $\vec{A}=a\hat{i}+\hat{j}+\hat{k}$, $\vec{B}=\hat{i}+b\hat{j}+\hat{k}$ and $\vec{C}=\hat{i}+\hat{j}+c\hat{k}$ are co-planar ,then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1, where\ \ a,b,c,\neq 1$

[OR]

If the vectors $\vec{a}=2\hat{i}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=4\hat{i}-3\hat{j}+7\hat{k}$, then find $\vec{r}$ such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}\circ\vec{a}=0$

[4]