



INDIAN SCHOOL SOHAR
SECOND TERM EXAM 2016 -2017
MATHEMATICS

Class: XII
Date: 22/11/16

Time: 3 Hrs
Marks: 100

General Instructions:

- a. All questions are compulsory.
- b. This question paper contains 29 questions.
- c. Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- d. Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- e. Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- f. Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION A

1. If $f: N \rightarrow R$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g: Q \rightarrow R$ be another function defined by $g(x) = x + 2$. Then find the value of $(g \circ f)\frac{3}{2}$.
2. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, satisfies the equation $A^2 - 3A - 7I = 0$ then find the value of A^{-1} .
3. Let A be the set of all positive real number and '*' be defined on A as $a * b = \frac{ab}{3}$ for all a, b belongs to A. Find the invertible elements of A.
4. If $|\vec{a}| = 10$ and $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$ then find the value of $|\vec{a} \times \vec{b}|$.

SECTION B

5. If $3 \tan^{-1} x + \cot^{-1} x = \pi$, then find the value of 'x'.
6. If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, then find the values of a, b and c
7. If $y = \tan^{-1} \frac{5x}{1-6x^2}$, $\frac{-1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$ then find $\frac{dy}{dx}$
8. Find the approximate value of $(1.999)^5$?

9. Evaluate $\int \frac{1}{e^x + e^{-x}} dx$

10. Find the differential equation of all non vertical lines in a plane

11. The vectors from origin to the points A and B are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ respectively, then find the area of the triangle OAB

12. If A and B are two events such that $P(A/B) = p$, $P(A) = p$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{9}$ then find the value of p.

SECTION C

13. Using matrices solve the equation $5x - 7y = 2$, $7x - 5y = 3$.

14. Discuss the continuity of the function f defined by $f(x) = |x-1| + |x-2|$ at $x = 1$ and $x = 2$

[OR]

For what choice of 'a' and 'b' is the function $f(x) = \begin{cases} x^2 & x \leq c \\ ax+b & x > c \end{cases}$ differentiable at $x = c$

15. If $y = \sin(m \sin^{-1} x)$, then prove that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0$

16. Find the equation of the tangent lines to the curve $y = 4x^3 - 3x + 5$ which is perpendicular to the line $9y + x + 3 = 0$.

[OR]

Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points on which the tangents are parallel to x axis

17. Find the largest possible area of a right angled triangle whose hypotenuse is 5 cm long. "Don't underestimate" comment the statement.

18. Evaluate $\int (3x+5) (\sqrt{2x^2 + 3x + 7}) dx$

[OR]

Evaluate $\int \frac{(3\sin\theta - 2)\cos\theta d\theta}{5 - \cos^2\theta - 4\sin\theta}$

19. Find the general solution of the differential equation $(1 + \tan y)(dx - dy) + 2x dy = 0$.

[OR]

Solve; $\frac{dy}{dx} - 3y = \sin 2x$

20. Find the value of λ so that the four points with the position vector $-\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} - \hat{k}$

$\hat{i} + \lambda\hat{j} + \hat{k}$ and $3\hat{j} + 3\hat{k}$ are co planar

21. Find the co ordinate of the foot of the perpendicular drawn from the point A (1,8,4) to the line

joining the points B(0 , - 1 , 3) and C (2 , -3 , -1).

22. Assume that the chances of a patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga? What is the biggest asset you would like to earn?

23. A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that at least one of the three marbles drawn be black ,if the first marble is red ?

SECTION D

24. Find the equation of the plane which contains the line $x = \frac{y-3}{2} = \frac{z-5}{3}$ and which is perpendicular to the plane $2x + 7y + 3z = 1$.

[OR]

Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to $(2, 3, -6)$

25. Show that the function $f : R \rightarrow \{x : x \in R : -1 < x < 1\}$ given by a function defined as

$$f(x) = \frac{x}{1 + |x|} \text{ is one to one and onto}$$

26. Using the properties of determinants, Prove that

$$\begin{vmatrix} a & b-c & b+c \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)(a^2 + b^2 + c^2)$$

27. Evaluate $\int_0^{\frac{3}{2}} |x \cos \pi x| dx$

[OR]

Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos \alpha \sin x}$

28. Sketch the graph of $f(x) = \begin{cases} |x-2| + 2 & \text{if } x \leq 2 \\ x^2 - 2 & \text{if } x > 2 \end{cases}$ and find the area of the region between 0 to 4

[OR]

Using integration find the area of the region bounded by the curves

$$\{(x, y) : 25x^2 + 9y^2 \leq 225 \text{ and } 5x + 3y \geq 15\}$$

29. An aero plane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passengers prefer to travel by second class then by first class. Determine how many tickets of each type must be sold to maximize profit for the airline. Form an LPP and solve it graphically. Explain the importance of simplicity in life.

*****The End*****