INDIAN SCHOOL SOHAR
PRE BOARD EXAMINATION - MATHEMATICS (041)
CLASS XII – 2016-17

Time allowed :**3** hours **Maximum Marks** : **100**
Date ; 15 / 01/ 2017

General Instructions:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

Section-A

Questions 1 to 4 carry 1 mark each.

1. Write the maximum number of equivalence relations on the set $\{1,2,3\}$?
2. If A is a square matrix of order 3 and $|A| = 7$, write the value of $|adj\ A|$
3. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, find the value of θ
4. If $*$ is a binary operation on the set **R**of real numbers defined by $a*b= a+b-2$, then find the identity element for the binary operation$*$.

Section-B

Questions 5 to 12 carry 2 marks each.

5. Evaluate $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$
6. If $A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ find the values of θ satisfying the equation $A + A^T = I_2$
7. Differentiate $\tan^{-1}\left\{\frac{\sqrt{1+x^2}+1}{x}\right\}$, $x \neq 0$ with respect to 'x'
8. Find the condition that the curves $2x = y^2$ and $2xy = k$ intersect orthogonally
9. Evaluate $\int_0^{\frac{\pi}{2}} e^{\sin x}\cos x\,dx$

10. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$

11. If $\left|\vec{a} + \vec{b}\right| = 60,\ \left|\vec{a} - \vec{b}\right| = 40,$ and $\left|\vec{a}\right| = 22,$ find $\left|\vec{b}\right|$,

12. If A and B are two events such that P (A) = $\frac{2}{5}$, P (B) = $\frac{1}{3}$ and P (A / B) = $\frac{1}{4}$,then find the value of P (A′ n B′)

Section-C

Questions 13 to 23 carry 4 marks each.

13. Form the differential equation having $y = \left(\sin^{-1} x\right)^2 + A\cos^{-1} x + B$ where A and B are arbitrary constants.

14. Solve : $\frac{dy}{dx} = \cos(x + y) + \sin(x + y)$

OR

Find the equation of the curve passing through (2 , 1) if the slope of the tangent to the curve at any point (x , y) is $\frac{x^2 + y^2}{2xy}$

15. The probability of a man hitting the target is 0. 25. He shoots 7 times. What is the probability of his hitting at least twice? .In your view , what are the values will improve the probability of success?

16. A bag contains 4 coins .Two coins are drawn at random and are found to be gold. What is the probability that the bag contains all gold coins?

OR

A letter is known to have either from TATANAGAR or from CALCUTTA. On the envelope two consecutive letters TA are available .What is the probability that the letter came from TATA NAGAR?

17. Show that the lines whose direction cosines are given by 2l + 2m – n = 0 and mn + nl + lm = 0 are at right angles

18. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a} \neq 0$, then prove that $\vec{b} = \vec{c}$

OR

Prove that for any three vectors $\vec{a}\ \vec{b}\ \vec{c}$: $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

19. Evaluate $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\ dx$

OR

Evaluate $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}\ dx$

20. An open box with the square base is to be made of a given quantity of cardboard of area c^2 .Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units."Past cannot be changed but we can change our future " comment on this statement

21. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

22. Find the value of p and q so that $f(x) = \begin{cases} x^2 + 3x + p & if\ x \le 1 \\ qx + 2 & if\ x > 1 \end{cases}$ is differentiable at x = 1

23. If $x = 3\cos\theta - 2\cos^3\theta$, $y = 3\sin\theta - 2\sin^3\theta$ prove that $\frac{dy}{dx} = \cot\theta$

Section-D

Questions 24 to 29 carry 6 marks each.

24. A company makes 3 types of calculator : A , B and C at factory I , and II. The company has orders for at least 6400 calculators of model A ,4000 calculators of model B and 4800 calculators of model C At Factory I 50 calculators of model A ,50 of model B and 30 of model C are made every day ,at factory II ,40 calculators of model A ,20 of model B and 40 of model C are made every day . It costs Rs 12,000 and Rs 15,000 each day to operate the factory I and II respectively .Find the number of days each factory should operate to minimize the operating cost and still meet the demand."Hard work " and "Smart work " differentiate.

25. A variable plane is at a constant distance p from the origin and meets the co ordinate axes in A , B , C .Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$.

OR

Prove that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ are co planar, and find the equation of the plane containing these lines.

26. Find the area of the region bounded by the line y = 3x + 2, the x axis and the co ordinates x = -1 and x = 1 .Also shade the region of the area.

OR

Find the area of the region bounded by the curve $y = x|x|$, the x axis and the co ordinates x = - 3 and x = 1 Also shade the region of the area.

27. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log \sin x \, dx$ **OR** Evaluate $\int_0^{2\pi} e^x \sin\left(\frac{\pi}{4}+\frac{x}{2}\right) dx$

28. Using properties of determinant prove that $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$ is divisible by a + b + c , and find the quotient.

29. Let A = N X N and let $*$ be a binary operation on A defined by
(a , b) $*$ (c , d) = (ad + bc , bd) , for all (a, b) , (c , d) $\in$ N X N . Show that
(i) '$*$' is commutative on A
(ii) '$*$' is associative on A
(iii) A has no identity element.