# INDIAN SCHOOL SOHAR TERM I EXAMINATION (2017-2018) MATHEMATICS 

Class: XI
Date: 24/09/2017
Time: 3 Hrs

## General Instructions

a. All questions are compulsory.
b. The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each and section $C$ comprises of 11 questions of 4 marks each. And section D comprises of 6 questions of six marks each.
c. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 3 questions of six mark each. You must attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

## SECTION - A

1. Simplify $\left[\left(A^{\prime} U B^{\prime}\right)-A\right]$
2. Find the domain of the function $f(x)=\frac{1}{\sqrt{|x|-x}}$. Explain with an example
3. For what real value of ' $x$ ' $\cos \theta=x+\frac{1}{x}$. justify
4. What is the sum of the series $i+i^{2}+i^{3}+\ldots---$ up to 1000 terms

## SECTION B

5. Two finite sets having ' $m$ ' and ' $n$ ' elements. The number of subsets of the first set is 112 more than that of the second set.Find the values of " $m$ " and " $n$ "
6. Redefine the function $f(x)=|x-2|+|x+2|$ in the interval $-3 \leq x \leq 3$
7. If $\tan \theta=\frac{a}{b}$ find the value of $b \cos 2 \theta+a \sin 2 \theta$
8. Find the principal argument of $(1+i \sqrt{3})^{2}$
9. Solve the following system of linear inequalities $\frac{2 x+1}{7 x-1}>5 \quad ; \frac{x+7}{x-8}>2$
10. How many even integers can be formed using the digits $2,3,4,5$ if repetition of digits is not allowed?
11. Find the co efficient of ' $x$ ' in the expansion of $\left(1-3 x+7 x^{2}\right)(1-x)^{16}$
12. Write the contra positive and converse of the statements "If 7 is greater than 5 , then 8 is greater than 6 "

## SECTION C

13. For all sets $A, B$ and $C$ Is $(A-B) \cap(C-B)=(A \cap C)-B$ ? Justify your answer.
14. Find the domain and range of the function $\mathrm{f}(\mathrm{x})=\left\{\begin{array}{lll}x+7 & \text { if }-3 \leq x \leq 5 \\ x^{2} & \text { if } 5 \leq x<7 \\ 6-2 x & \text { if } & x \geq 7\end{array}\right\}$

## OR

Find the domain and range of the function $\mathrm{f}(\mathrm{x})=\frac{1}{(2 x-3)(x+1)}$
15. Solve the equations $\tan \theta+\tan 2 \theta+\sqrt{3} \tan \theta \tan 2 \theta=\sqrt{3}$
16. Find the value of $3\left[\sin ^{4}\left(\frac{3 \pi}{2}-\alpha\right)+\sin ^{4}(3 \pi+\alpha)\right]-2\left[\sin ^{6}\left(\frac{\pi}{2}+\alpha\right)+\sin ^{6}(5 \pi-\alpha)\right]$

## OR

If $\cos (\alpha+\beta)=\frac{4}{5}, \sin (\alpha-\beta)=\frac{5}{13}$ and $\alpha$ and $\beta$ lie between 0 and $\frac{\pi}{4}$ find the value of $\tan 2 \alpha$
17. A sequence $a_{1}, a_{2}, a_{3} \ldots \ldots$ is defined by letting $a_{1}=3, a_{k}=7 a_{k-1}$ for all natural numbers $k>1$. Show that $a_{n}=37^{n-1}$ for all-natural numbers.
18. If $\mathrm{a}=\cos \theta+i \sin \theta$ find the value of $\frac{1+a}{1-a}$

## OR

Find the square root of a complex number 1 - i
19. Find the real value of $\theta$ for which the expression $\frac{1+i \cos \theta}{1-2 i \cos \theta}$ is a real number
20. A solution of $8 \%$ boric acid is to be diluted by adding a $2 \%$ boric acid solution to it. The resulting mixture is to be more than $4 \%$ but less than $6 \%$ boric acid. If we have 640 litres of the $8 \%$ solution, how many litres of the $2 \%$ solution will have to be added?
21. If a convex polygon has 44 diagonals, find the number of its sides.

## OR

If the letters of the word "SCHOOL" is arranged as per the dictionary, find the number of the word SCHOOL.
22. Find the values of ' $x$ ' if the middle term of the $\left(\frac{1}{x}+x \sin x\right)^{10}$ is $\frac{63}{8}$
23. Prove by direct method that for any integer $n, n^{3}-n$ is always even integer

## SECTION D

24. In a survey of 100 students the number of students studying various languages were found to be, English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no languages 24. Find the number of students studying at least two languages. Write advantages of learning many languages.

## OR

Out of 1020 boys in a school, 406 play crickets, 324 play hockey, 250 play football, 80 play cricket and hockey, 64 play hockey and football, 92 play cricket and football, 246 play none of these games. How many play all the three games? "Exams verses every day games" express your views.
25. Find the common solution region of the following system of inequalities graphically:$3 x+y \geq 4 ; x+5 y \geq 4 ; x+y \leq 4 ; x \leq 3, y \leq 3, x \geq 0 ; y \geq 0$
26. If $\alpha$ and $\beta$ are two distinct roots of the equation $\mathrm{a} \cos \theta+\mathrm{b} \sin \theta=\mathrm{c}$ then shows that

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\cos (\alpha-\beta)=\frac{2 c^{2}-\left(a^{2}+b^{2}\right)}{a^{2}+b^{2}} \text { and } \cos (\alpha+\beta)=\frac{a^{2}-b^{2}}{a^{2}+b^{2}}
$$

27. Find the co efficient of $x^{7}$ in $\left(a x^{2}+\frac{1}{b x}\right)^{11}$ and $x^{-7}$ in $\left(a x+\frac{1}{b x^{2}}\right)^{11}$ and find the relation between a and b so that the co efficient are equal.
28. Prove that $(2 \cos \theta-1)(2 \cos 2 \theta-1)(2 \cos 4 \theta-1) \ldots \ldots \ldots \ldots . .\left(2 \cos 2^{n-1} \theta-1\right)-\frac{2 \cos 2^{n} \theta+1}{2 \cos \theta+1}, n \in N$ Using principle of mathematical induction.

## OR

Prove that $\sin \theta+\sin 2 \theta+\sin 3 \theta \ldots \ldots \ldots \ldots \ldots \ldots \ldots+\sin n \theta=\frac{\sin \left(\frac{n+1}{2}\right) \theta \sin \frac{n \theta}{2}}{\sin \frac{\theta}{2}}, n \in N$
Using principle of mathematical induction.
29. In triangle $A B C$ prove that $\cos 2 A+\cos 2 B+\cos 2 C=-1-4 \cos A \cos B \cos C$

## OR

In any triangle $\mathrm{ABC} \frac{b+c}{12}=\frac{c+a}{13}=\frac{a+b}{15}$ then prove that $\frac{\cos A}{2}=\frac{\cos B}{7}=\frac{\cos C}{11}$

