INDIAN SCHOOL SOHAR
TERM I EXAMINATION (2017 -2018)
MATHEMATICS

Class: XII **Time: 3 Hrs**
Date: 12/09/2017 **Max. Marks: 100**

General Instructions

a. All questions are compulsory.

b. The question paper consist of 29 questions divided into four sections A, B,C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each and section C comprises of 11 questions of four marks each. And section D comprises of 6 questions of six marks each.

c. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

d. There is no overall choice. However, internal choice has been provided in 03 questions of four marks each and 03 questions of six mark each. You have to attempt only one of the alternatives in all such questions.

e. Use of calculators is not permitted.

SECTION – A

1. Write the inverse of the following matrices, if it exists $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$

2. Find the values of k so that the function f is continuous at x=0 $\begin{Bmatrix} \frac{\sin 5x}{3x} + \cos x & if \ x \neq 0 \\ k & if \ x = 0 \end{Bmatrix}$

3. Evaluate $\int x^3 \cos x^4 \, dx$

4. Find the cartesian equation of the plane passing through the point (1,2,3) and is perpendicular to the line $\frac{x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$

SECTION – B

5. If the product of the matrix B = $\begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ with a matrix A has inverse C = $\begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$ then find the inverse of A.

6. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When x =10cm and y = 6cm, find the rates of change of

(a) the perimeter and (b) the area of the rectangle.

7. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$

8. Find the intervals in which the function f given by f (x) = sin x + cos x, strictly increasing or strictly decreasing

9. Find the co ordinates of the foot of the perpendicular drawn from the point (3,2,1) to the plane $2x - y + z + 1 = 0$.

10. Prove that $\tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$

11. A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. Formulate the problem so as to maximize the total profit of the society?

12. Evaluate $\int \frac{dx}{\cos x \sqrt{\cos 2x}} dx$

SECTION C

13. If $(\sin^{-1}x)^2 + (\cos^{-1}x)^2 = \frac{5\pi^2}{8}$ find the value of 'x'

[OR]

If $x \in \left[\frac{\pi}{2}, \pi\right]$ then find the value of $\cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

14. If $\alpha, \beta\ and\ \gamma$ are the roots of the equation $x^3 + ax^2 + b = 0$, then using the properties of determinants find the value of $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

15. If $y = \tan^{-1}\frac{1}{1+x+x^2} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + ...$ up to n terms. Find the value of $y_1(0)$.

16. Evaluate $\int e^{3x} \sin 4x \, dx$

[OR]

Evaluate $\int \frac{\sin 2x}{(1+\sin x)\,(2+\sin x)} \, dx$

17. Evaluate $\int \frac{dx}{\cos(x+a)\,\cos(x+b)}$

[OR]

Evaluate $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

18. The vector $\vec{a} = \alpha\,\hat{i} + 2\,\hat{j} + \beta\,\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$ .Find the value of α *and* β

19. If the four points a,b,c,d are co planar then find the value of [b, c, d] +[c,a,d]+[a,b,d]

20. Find the equation of a line through the point (-1,3,-2) and perpendicular to the lines

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

21. Find the approximate volume of metal in a hallow spherical shell whose internal and external radii are 3 cm and 3.0005 cm respectively. A small increment in radius consumes large amount of material. Interpret your understanding in terms practical life.

[OR]

The volume of a cube increases at a constant rate .Prove that the increase in its surface area varies inversely as the length of the side .Compare the rate of learning in various age level and what do you infer from that?

22. Find the value of "k "in the function $f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x} & if \;\; x \neq 0 \\ \frac{1}{2} & if \;\; x = 0 \end{cases}$ is continuous at x = 0

23. One kind of cake requires 200 g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage for other ingredients used in making the cakes. Formulate the other as the linear programming problem and solve graphically. To obtain the target what do you expect? Large number of source or smart utilization of source, explains.

SECTION D

24. Solve the system of equation $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$; $\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$; $\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

25. Given a non-empty set X, let $*$: P(X) × P(X) → P(X) be defined as

A * B = (A – B) ∪ (B – A), ∀ A, B ∈ P(X). Show that the empty set φ is the Identity for the operation $*$ and all the elements A of P(X) are invertible with $A^{-1} = A$.

[OR]

Let f, g: **R** → **R** be two functions defined as $f(x) = x + |x|$ and $g(x) = x - |x|$ ∀ $x \in$ **R**. Then, find f *o g* and *g o f*.

26. The sum of the surface areas of a rectangular parallelepiped with sides x, $2x$ *and* $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes.

[OR]

Find the shortest distance of the point (0, c) from the parabola $y = x^2$, where $0 \le c \le 5$.

27. Show that the equation of normal at any point on the curve $x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$ is $4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$.

28. Find the equation of the plane passing through the line of intersection of the planes x-2y+z =1 and 2x+y+z = 8 and parallel to the line with the direction ratios 1, 2, 1. Also find the perpendicular distance of the point (3, 1, 2) from this plane.

29. Evaluate $\int \frac{x^2}{x^4-1}dx$ **[OR]** Evaluate $\int \frac{x+2}{2x^2+6x+5}dx$