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INDIAN SCHOOL SOHAR
TERM II EXAMINATION (2017 -2018)
MATHEMATICS

Class: XII **Time: 3 Hrs**
Date: 30/11/2017 **Max. Marks: 100**

General Instructions

a. *All questions are compulsory.*
b. *The question paper consist of 29 questions divided into four sections A, B,C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each and section C comprises of 11 questions of four marks each. And section D comprises of 6 questions of six marks each.*
c. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
d. *There is no overall choice. However, internal choice has been provided in 03 questions of four marks each and 03 questions of six mark each. You have to attempt only one of the alternatives in all such questions.*
e. *Use of calculators is not permitted.*

SECTION A

1. If A is a square matrix such that $A^2 = A$, then find the value of $(I + A)^3 - 7A$
2. Find the point on the curve xy + 4 = 0 which the tangent is inclined at an angle of 45^o with x axis
3. Evaluate $\int \frac{1+\cot x}{1-\cot x}\,dx$
4. find the volume of the tetrahedron , whose three co terminus edges are $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $3\hat{i}-\hat{k}$

SECTION B

5. Evaluate $\int \frac{3\left(e^{2x}+e^{4x}\right)}{e^{x}+e^{-x}}\,dx$
6. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportaion cost per quintal		
From / To	**A**	**B**
D	**6**	**4**
E	**3**	**2**
F	**2.5**	**3**

Write down the equation to find the cost of the transportation.

7. A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. If A starts the game, find the probability of winning the game by A in third throw of a pair of dice.
8. A straight line makes angles 60^o and 45^o with the positive direction of X axis and Y axis respectively. What angle does it make with the Z axis?
9. Using Rolles theorem find the point on the curve y = x (x - 4), $x \in [0, 4]$ where the tangent is parallel to the x axis.
10. If A = $\begin{bmatrix} \alpha & \beta \\ \chi & -\alpha \end{bmatrix}$ where A^2 = I, find the value of $\alpha^2 + \beta\chi$.
11. Find the rate of change of the curved surface of a right circular cone of radius r and height h with respect to change in radius.

SECTION C

12. The probability distribution of a random variable X is given as follows

$$P(X = x) = \begin{cases} k\,x^2 & for\ x = 1, 2, 3 \\ 2kx & for\ x = 4, 5, 6 \\ 0 & otherwise \end{cases}$$ Find the mean and variance of the random variable.

13. Find the image of the point (1,2,3) in the plane x+2y+4z = 38.
14. If $\vec{a} = 2\hat{i} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$, find a vector $\vec{r}$ which satisfies $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \circ \vec{a} = 0$.
15. Solve for x $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$.

[OR]

Show that $2\tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right] = \tan^{-1}\left(\frac{\cos\alpha\cos\beta}{\sin\alpha + \sin\beta}\right)$

16. Using the properties of determinant prove that $\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} (x - p)\left(x^2 + px - 2q^2\right)$
17. If y = (sinx)^x + (cosx)$^{\tan x}$ find $\frac{dy}{dx}$.

[OR]

If $\sqrt{1 - x^6} + \sqrt{1 - y^6} = a\left(x^3 - y^3\right)$, Prove that $\frac{dy}{dx} = \frac{x^2\sqrt{1 - y^6}}{y^2\sqrt{1 - x^6}}$

18. Evaluate $\int_0^{\pi} \frac{dx}{5+\cos x}$

[OR]

Evaluate $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

19. If a young man rides his motor-cycle at 25 km per hour, he had to spend Rs 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to Rs 5 per km and rate of pollution also increases. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour?. Express this problem as an L.P.P. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?

20. Evaluate $\int \frac{x^2+1}{x^4+x^2+1} dx$

21. show that the area of the parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ = $\frac{1}{2}\left|\vec{a} \times \vec{b}\right|$.Also find the area of the parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$

22. Find the particular solution of the following differential equation ($\tan^{-1}y - x$) dy = $(1+y^2)$ dx given that x = 0 , y = 0

23. Show that the relation R on the set N X N defined by (a, b) R (c , d) iff ad (b + c) = bc (a + d) is an equivalence relation

SECTION C

24. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance ? Is regularity required only in school ? Justify your answer.

25. Find the area bounded by the lines x + 2y = 2, y – x = 1, 2x + y = 7

[OR]

Using integration find the area of the region $\left\{(x, y) : |x-1| \le y \le \sqrt{5-x^2}\right\}$

26. Solve the differential equation $3e^x \tan y\, dx + (2 - e^x)\sec^2 y\, dy = 0$ given that y = $\frac{\pi}{4}$ when x = 0

27. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs 300/- per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Re 1/- one subscriber will discontinue the service. Find what increase will bring maximum profit?

[OR]

An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$

28. Determine the three planes through the intersection of the planes x + y + z = 1, and 2x + 3y-z + 4 = 0 which are parallel to the three co ordinate axes .Also find the equation of the plane perpendicular to YZ plane passing through the point (2,1,1) and (4 , -5 , 3)

[OR]

A variable plane is at a constant distance 'p'from the origin and meet the axes in A , B , C .Show that the locus of the centreoid of the tetrahedron OABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$

29. An amount of Rs 50000 is put in to three investments at the rate of interests 6% , 7% and 8% per annum respectively. the total annual income is Rs 3580.If he combined income from the first two investments is Rs 700 more than the income from the third, find the amount of each investment by matrix method.