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INDIAN SCHOOL SOHAR
PRE BOARD EXAMINATION (2017 -2018)
MATHEMATICS

Class: XII **Time: 3 Hrs**
Date: 09 /01/2018 **Max. Marks: 100**

General Instructions

a. All questions are compulsory.
b. The question paper consist of 29 questions divided into four sections A, B,C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each and section C comprises of 11 questions of four marks each. And section D comprises of 6 questions of six marks each.
c. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d. There is no overall choice. However, internal choice has been provided in 03 questions of four marks each and 03 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

SECTION A

1. The foot of the perpendicular drawn from the origin to a plane is (5, -3 , -2). Find the equation of the plane.
2. Evaluate $\int_0^{\pi} x \sin x \cos^2 x \, dx$
3. Find the maximum value of f(x) = sinx + $\sqrt{3}\cos x$
4. Find the value of the operation $R_1 \to R_1 - 3R_2$ in the matrix equation

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

SECTION B

5. Evaluate $\int \frac{2x-1}{2x+3} dx$
6. Find the corner points of the feasible region and the maximum value of the function Z = 4x + 3y under the constraints $x \geq 2$, $y \geq 3$, $x \geq 0$ *and* $y \geq 0$

7. A box has 5 blue and 4 red balls .One ball is drawn at random and not replaced .Its colour is also not noted .Then another ball is drawn at random .What is the probability of the second ball being blue .

8. Differentiate $\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$ with respect to x .

9. Prove that the curves xy = 4 and $x^2 + y^2 = 8$ touch each other .

10. Prove that $\left|\vec{a}\times\vec{b}\right|^2+\left(\vec{a}\cdot\vec{b}\right)^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$

11. Find the value of α If A = $\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ *and* $A^{-1}=A^1$

12. A swimming pool is to be drained for cleaning. If L represents the litres of water in the pool t seconds ,after the pool has been plugged off to drain and. $L = 200(10 - t^2)$..How fast is the water running out at the end of 5 s and what is the average rate at which the water flows out during the first 5 s?

SECTION C

13. Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution.

14. The vectors $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}-2\hat{k}$ are the adjacent sides of a parallelogram .Find the angle between the diagonal.

15. Evaluate $\int_0^{\pi}\frac{x}{1+\sin x}\,dx$ **OR** Evaluate $\int_1^2\frac{dx}{\sqrt{(x-1)(x-2)}}\,dx$

16. Suppose you have two coins which appear identical in your pocket. You know that one is fair and one is two headed, If you take one out, toss it and get a head. What is the probability that it was a fair coin.

17. Evaluate $\int\frac{x^{\frac{1}{2}}}{1+x^{\frac{3}{4}}}\,dx$ **OR** Evaluate $\int\frac{\sin^{-1}x}{\left(1-x^2\right)^{\frac{3}{4}}}\,dx$

18. If x = sint and y = sin pt then prove that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

OR

Differentiate $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ with respect to $\tan^{-1}x$

19. Using the properties of determinants prove that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+1 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

20. A manufacturer of electronic circuits has a stock of 200 resistors ,120 transistors ,150 capacitors and is required to produce two types of circuits A and B .Type A requires 20 resistors 10 transistors and 10 capacitors .Type B requires 10 resistors 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B is Rs 60 ,formulate this problem as LPP so that the manufacturer can mazimise his profit.

21. Find the equation of a curve passing through the origin and satisfying the differential equations $\left(1+x^2\right)\frac{dy}{dx} + 2xy = 4x^2$

22. Prove that $\cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right) = 7$

23. Using Vectors find the value of k , such that the points (k , - 10 , 3) , (1 , -1 , 3) and (3 , 5 , 3) are collinear.

SECTION D

24. Draw a rough sketch of the region $\{(x,y): y^2 \le 6ax \text{ and } x^2 + y^2 \le 16a^2\}$ .Also find the area of the region sketched using integration.

OR

Find the area of the region bounded by the curve $y^2 = 2x$ and $x^2 + y^2 = 4x$

25. A metal box with the square base and vertical sides is to contain 1024 cm^3.If the material for the top and bottom costs Rs 5 / cm^2 and the material for the sides costs Rs 2.5 / cm^2 .Find the least cost of the box. “Volumes can remain same even the surface areas are different” writhe the value you understand from these statements,

26. If the plane ax + by = 0 is rotated about its line of intersection with the plane z = 0 through an angle α, then prove that the equation of the plane in its new position is $ax + by \pm \left(\sqrt{a^2 + b^2}\ \tan\alpha\right) z = 0$

OR

Show that the straight lines whose direction cosines are given by 2l + 2m –n = 0 and mn + nl + lm = 0 are at right angles

27. Find the general solution of (1 + tany) (dx – dy) +2x dy= 0

OR

Find the equation of the curve passing through (1, 1) .If the tangent drawn at any point P(x , y) on the curve meets the co ordinate axes at A and B such that P is the midpoint of AB

28. Using matrix method solve the system of linear equations x – 2y = 10 , 2x – y – z = 8 and -2y +z = 7

29. Let A = {1, 2, 3, ... 9} and R be the relation in A XA defined by (*a*, *b*) R (*c*, *d*) if $a + d = b + c$ for (a, b) , (c , d) in A X A. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)].