INDIAN SCHOOL SOHAR FINAL EXAMINATION (2017-2018)

MATHEMATICS
Class: XI
Date: 04/03/2018

Time: 3 Hrs.
Max. Marks: 100

## General Instructions

a. All questions are compulsory.
b. The question paper consist of 29 questions divided into four sections $A, B, C$ and $D$. Section $A$ comprises of 4 questions of one mark each, section $B$ comprises of 8 questions of two marks each and section C comprises of 11 questions of four marks each. And section $D$ comprises of 6 questions of six marks each.
c. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d. There is no overall choice. However, internal choice has been provided in 03 questions of four marks each and 03 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

## SECTION A

1. If $f(x)=x^{3}-\frac{1}{x^{3}}$, find the value of $f(x)+f\left(\frac{1}{x}\right)$
2. What is the value of $\frac{i^{4 n+1}-i^{4 n-1}}{2}$ ?
3. The centroid of the triangle ABC is at the point $(1,1,1)$. If the co ordinates of point A and B are ( $3,-5,7$ ) and ( $-1,7,-6$ ) respectively, find the co ordinate of the point $C$.
4. Identify the type of "OR "used in the following statement, and check whether the statement is true or false "To enter a public library children need an identity card from the school or a letter from the school authorities".

## SECTION B

5. Find the principal value of $x$ satisfying the equation $2 \cos ^{2} x+3 \sin x=0$
6. Find the modulus and argument of the complex number $-\sqrt{3}+i$
7. In how many ways can the letters of the word PERMUTATIONS be arranged if there are always 4 letters between $P$ and $S$ ?
8. If the line joining two points $\mathrm{A}(2,0)$ and $\mathrm{B}(3,1)$ is rotated about A in anticlockwise direction through an angle $15^{\circ}$.Find the equation of the line in the new position..
9. Find the equation of the ellipse which passes through the point $(-3,1)$ and has eccentricity $\frac{\sqrt{2}}{5}$ with $x$ axis as its major axis and centre at the origin.
10. Find the equation of the set of points $P$ such that the distances from the point $A(3,4,-5)$ and $B(-2,1,4)$ are equal.
11. Find the derivative of $\frac{x+\cos x}{\tan x}$
12. A typical PIN (Personal identification number) is a sequence of any four symbols chosen from 26 letters in the alphabet and the ten digits. If all PIN s are equally likely, what is the probability that a randomly chosen PIN contains repeated symbol?

## SECTION C

13. Let A, B and C be non-empty sets. Using the properties of sets prove that $A \cup(B \cap C)=(A \bigcup B) \cap(A \cup C)$.
14. Find the domain and range of the real valued function $f(x)=\frac{x^{2}}{1+x^{2}}$
15. Find the value of $\sin 20^{\circ} \sin 40^{\circ} \sin 60^{\circ} \sin 80^{\circ}$

## OR

If $\sin \theta+\cos \theta=1$, find the general value of $\theta$
16. Using Principle of mathematical induction prove that $2.7^{\mathrm{n}}+3.5^{\mathrm{n}}-5$ is divisible by 24 for all $\mathrm{n} \in \mathrm{N}$
17. If $(x+i y)^{3}=u+i v$, then show that $\frac{u}{x}+\frac{v}{y}=4\left(x^{2}-y^{2}\right)$
18. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these
(i) Four cards are of the same suit,
(ii) Four cards belong to four different suits,
(iv)Two are red cards and two are black cards,

## OR

Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the $50^{\text {th }}$ word?
19. If $S_{1}, S_{2}, S_{3}$ are the sum of first $n$ natural numbers, their squares and their cubes, respectively, show that
$9 \quad S_{2}^{2}=\mathrm{S}_{3}\left(1+8 \mathrm{~S}_{1}\right)$.
20. Find the sum to $n$ terms of the series: $5+11+19+29+41 \ldots$
21. If $P$ is the length of the perpendicular from the origin to the line whose intercepts on the axes are a and b , then show that $\frac{1}{p^{2}}=\frac{1}{a^{2}}+\frac{1}{b^{2}}$
22. Evaluate $\underset{x \rightarrow 0}{L t} \frac{\cos a x-\cos b x}{\cos c x-1}$
23. On her vacations Veena visits four cities (A, B, C and D) in a random order. What is the probability that she visits
(i) A before B and B before C ?
(ii) A first and B last?
(iii) A either first or second?
(iv) A just before B?

## SECTION D

24. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects. Define the term "Education "in your view.
25. Prove that $\cos ^{2} x+\cos ^{2}\left(x+\frac{\pi}{3}\right)+\cos ^{2}\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$.

## OR

Derive $\cos (x+y)=\cos x \cos y-\sin x \sin y$ using unit circle and deduce the condition to find $\sin (x-y)$
26. Graphically solve the system of linear inequality $4 x+3 y \leq 60, y \geq 2 x, x \geq 3, x, y \geq 0$
27. Find $a, b$ and $n$ in the expansion of $(a+b)^{\mathrm{n}}$ if the first three terms of the expansion are 729,7290 and 30375, respectively.

## OR

Find $n$, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of

$$
\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^{n} \text { is } \sqrt{6}: 1
$$

28. Find the mean, variance and standard deviation of the following data

| Class | $0-30$ | $30-60$ | $60-90$ | $90-120$ | $120-150$ | $150-180$ | $180-210$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No of items | 2 | 3 | 5 | 10 | 3 | 5 | 2 |

29. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m . Find the length of a supporting wire attached to the roadway 18 m from the middle.

## OR

Find the equation of the circle which passes through the points $(20,3),(19,8)$ and $(2,-9)$. Find its centre and radius.

