INDIAN SCHOOL SOHAR
TERM I EXAMINATION (2018 -19)
SUBJECT: MATHEMATICS

CLASS: XII **MAX. MARKS: 100**
DATE: 23 /09/2018 **DURATION: 3 HOURS**

General Instructions

a. *All questions are compulsory.*
b. *The question paper consist of 29 questions divided into four sections A, B,C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each and section C comprises of 11 questions of four marks each. And section D comprises of 6 questions of six marks each.*
c. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
d. *There is no overall choice. However, internal choice has been provided in 03 questions of four marks each and 03 questions of six mark each. You have to attempt only one of the alternatives in all such questions.*
e. *Use of calculators is not permitted.*

1. Define $\sin^{-1}x - \sin^{-1}y$ when $x > 0$, $y < 0$ and $x^2 + y^2 = 1$

2. Write the differentiation of $\sin^2 x$ with respect to $e^{\cos x}$

3. Evaluate $\int 5x^4 \sqrt{x^5 + 1}\, dx$

4. Find the value of the expression $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a} \cdot \vec{b}\right)^2$

SECTION B

5. Consider the function $f \left[0 , \frac{\pi}{2}\right] \to R$ given by f(x) = sinx, and $g \left[0 , \frac{\pi}{2}\right] \to R$ by g (x) = cosx, show that f and g are one to one and f + g is not one to one.

6. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right)$ $|x| > 1$ in the simplest form.

7. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = 0$

8. Show that the points (a , b + c) , (b , a + c) , (c , a + b) are collinear using determinants.

9. Discuss the continuity of the function defined by $f(x) = \begin{cases} x+2 & if\ x \le 1 \\ x-2 & if\ x > 1 \end{cases}$ using the graph of the function.

10. The total cost of C(x) in Rupees associated with the production of x units of an item is given by C(x) = 0.007 x^3 – 0.003x^2 +15x + 4000. Find the marginal cost when 17 units are produced.

11. Let θ be the angle between the unit vectors $\hat{a}$ and $\hat{b}$. For what value of θ, $\hat{a} + \hat{b}$ is a unit vector?

12. If the foot of the perpendicular drawn from the origin to a plane is (5, -3, -2), find the equation of the plane.

SECTION C

13. Consider f : {1, 2, 3} → {a, b, c} and g : {a, b, c} → {apple, ball, cat} defined as f (1) = a, f(2) = b, f(3) =c, g(a) = apple, g(b) = ball and g(c) = cat. Show that f, g and gof are invertible. Find out f^{-1}, g^{-1} and $(gof)^{-1}$ and show that $(gof)^{-1} = f^{-1} o\, g^{-1}$.

14. Solve for x and y $\tan^{-1} x + \cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right) = \sin^{-1}\frac{3}{\sqrt{10}}$

15. Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ show that $(a I + b A)^n = a^n I + n\, a^{n-1} b\, A$ where I is the identity matrix of order 2 and $n \in N$

[OR]

Obtain the inverse of the matrix using elementary operation A = $\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$

16. if $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix}$ = 0 ,find the value of θ lying between 0 and $\frac{\pi}{2}$,using the properties of determinants.

17. Show that f(x) = $|x-5|$ is continuous but not differentiable at x = 5.

[OR]

Find the value of the following function if f(x) = $\begin{bmatrix} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} & if\ -1 \le x < 0 \\ \frac{2x+1}{x-1} & if\ 0 \le x \le 1 \end{bmatrix}$

is continuous at x = 0

18. If y = $e^{a\cos^{-1}x}$ prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

19. Two men A and B start with the velocities V at the same time from the junction of two roads inclined at 45° to each other. If they travel by different roads, find the rate at which they are being separated.

20. Evaluate $\int \sqrt{\tan x} + \sqrt{\cot x}\ dx$

[OR]

Evaluate $\int \frac{(3\sin\theta - 2)\cos\vartheta}{5 - \cos^2\theta - 4\sin\theta}\ d\theta$

21. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x}\ dx$

22. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$

23. Find the angle between the lines whose directions are given by the equations l + m + n = 0 and $l^2 + m^2 + n^2 = 0$

[OR}

Find the equations of two lines through the origin which intersects the lines $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angles of $\frac{\pi}{3}$ each.

SECTION – D

24. If A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ then find BA and use this to solve the equation y + 2z = 7, x – y = 3 and 2x + 3y + 4z = 17

25. A window is in the form of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 16 m, find the dimensions of the window so that maximum light can be admitted.

[OR]

If the sum of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$

26. Evaluate $\int_0^1 e^{2-3x}\, dx$ as a limit of a sum

27. Evaluate $\int \sqrt{\tan x}\, dx$

[OR]

Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)^2}$

28. Find the equation of the plane perpendicular to the YZ plane and passing through the points

(1, -2 , 4) and (3 , 4 -5)

[OR]

$\vec{AB} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are two vectors .The position vectors of $\vec{A}$ and $\vec{C}$ are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{i} + 2\hat{k}$ respectively . Find the position vector of P on the line AB and a point Q on the line CD such that $\vec{PQ}$ is perpendicular to both $\vec{AB}$ and $\vec{CD}$

29. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labor hours for fabricating and 1 labor hour for finishing. Each piece of Model B requires 12 labor hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realize a maximum profit? What is the maximum profit per week?

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