INDIAN SCHOOL SOHAR
TERM II EXAMINATION (2018 -2019)
MATHEMATICS

Class: XII   **Time: 3 Hrs**
Date: 28/11/2018   **Max. Marks: 100**

General Instructions:
1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Questions 1 – 4 in Section A are very short-answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 marks each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 marks each.
6 Questions 24 – 29 in Section D are long-answer II type questions carrying 6 marks each.

_____

SECTION A

1. If $A = [\ a_{ij}\ ]$ is a matrix of order 2 X 2 , $|A| = -15$ and c_{ij} represent the co factor of a_{ij}, then

   find $a_{21}\ c_{21} + a_{22}\ c_{22}$.

2. Find the derivative of $f(\ e^{\tan x})$ with respect to x at $x = 0$, it is given that $f'(1) = 5$.

3. Find the differential equation of the family of curves $y^2 = 4ax$

4. Find the acute angle which the line with the direction cosines $\dfrac{1}{\sqrt{3}}$, $\dfrac{1}{\sqrt{6}}$, n makes with the positive

   direction of z axis

   [OR]

   Find the direction cosines of the line $\dfrac{x-1}{2} = -y = \dfrac{z+1}{2}$

SECTION B

5. How many equivalence relations on the set (1,2,3) containing (1 , 2) and (2 , 1) are there in all ?. Justify tour answer.

6. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2\ A^{-1} = 9\ I - A$

7. Find $\displaystyle\int \dfrac{(x^2 + \sin^2 x)\sec^2 x}{1 + x^2}\ dx$

8. Find $\displaystyle\int \dfrac{3 - 5\sin x}{\cos^2 x}\ dx$

9. Find the sum of the order and degree of the following differential equation $\dfrac{d^2 y}{dx^2} + \sqrt[3]{\dfrac{dy}{dx}} + (1 + x) = 0$

10. If $\vec{a} + \vec{b} + \vec{c} = 0$ and $\left|\vec{a}\right| = 5$, $\left|\vec{b}\right| = 6$ and $\left|\vec{c}\right| = 9$, then find the angle between $\vec{a}$ and $\vec{b}$

[OR]

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a} \circ \vec{b} = \vec{a} \circ \vec{c} = 0$, and the angle between , $\vec{b}$ *and* $\vec{c}$ is $\dfrac{\pi}{6}$,

then prove that $\vec{a} = \pm 2 \left(\vec{b} \times \vec{c} \right)$

11. Evaluate $P (A \cup B)$, if $2 P (A) = P (B) = \dfrac{5}{13}$ and $P (A / B) = \dfrac{2}{5}$.

12. A problem in mathematics is given to 4 students A , B , C , D , the chances of solving the problem
respectively $\dfrac{1}{3}$, $\dfrac{1}{4}$, $\dfrac{1}{5}$ and $\dfrac{2}{3}$. What is the probability that the problem will be solved ?.

[OR]

A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

SECTION C

13. If the function f: R $\rightarrow$R be defined by $f(x) = 2x - 3$ and g : R $\rightarrow$R be defined by $g(x) = x^3 + 5$, then find $f \circ g$ and prove that $f \circ g$ invertible .

[OR]

A binary operation ' $*$ ' defined on the set R of real number by $a * b = \begin{cases} a , & if\ b = 0 \\ |a| + b , & if\ b \neq 0 \end{cases}$ ' if atleast

one of is 0 , then prove that $a * b = b * a$, check whether $*$ is commutative . Find the identity element for $*$, if it exist .

14. Does the following trigonometric equation have any solutions? .If yes obtain the solution(s)

$\tan^{-1} \left(\dfrac{x+1}{x-1} \right) + \tan^{-1} \left(\dfrac{x-1}{x} \right) = - \tan^{-1} 7$

15. Using the properties of determinant prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3\ a\ b\ c\ -\ a^3 - b^3\ -\ c^3$

16. Find all points of discontinuity of f, where f is defined by $\begin{cases} |x| + 3 & if\ x \leq -3 \\ -2x & if\ -3 < x < 3 \\ 6x + 2 & if\ x \geq 3 \end{cases}$

[OR]

Find the values of k so that the function f(x) is continuous at x $= \dfrac{\pi}{2}$,where

$$f(x) = \begin{cases} \dfrac{k \cos x}{\pi - 2x} & if \quad x \ne \dfrac{\pi}{2} \\ \\ 3 & if \quad x = \dfrac{\pi}{2} \end{cases}$$

17. If $y = \cos(m \cos^{-1}x)$, show that $(1 - x^2) y_2 - x y_1 + m^2 y = 0$

18. Find the equation of the tangent to the curve $y = \cos(x + y)$, $-2\pi \le x \le 0$, that is parallel to the line $x + 2y = 0$

19. Find $\int (x - 4) \sqrt{4 + 3x - x^2} \ dx$

20. Evaluate $\displaystyle\int_{-1}^{1} \dfrac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$

21. Solve the differential equation $(x^2 - y^2) dx + 2xy \, dy = 0$

   [OR]

   Find the particular solution of the differential equation $(1 + x^2) \dfrac{dy}{dx} + 2xy = \dfrac{1}{1 + x^2}$

   given that $y = 0$ when $x = 1$

22. If the vectors $\vec{p} = a\hat{i} + \hat{j} + \hat{k}$, $\vec{q} = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + c\hat{k}$ are co planar, then for

   $a, b, c \ne 1$ show that $\dfrac{1}{1-a} + \dfrac{1}{1-b} + \dfrac{1}{1-c} = 1$

23. Find the equation of the line which intersects the lines $\dfrac{x+2}{1} = \dfrac{y-3}{2} = \dfrac{z+1}{4}$ and $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$

   and passes through the point $(1, 1, 1)$

SECTION D

24. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the system of equations $3x + 4y + 7z = 14$,

   $2x - y + 3z = 4$, $x + 2y - 3z = 0$

   [OR]

If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, find the inverse of A using elementary transformation. and hence solve the

matrix equation $XA = [\ 1\ \ 0\ \ 1\ ]$

25. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

26. Using integration find the area in the first quadrant bounded by the curve $y = x|x|$, the circle

$x^2 + y^2 = 2$ and the y axis

[OR]

Using integration find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

27. Find the equation of the plane which contains the line of intersection of the planes

$x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and whose x intercept is twice its z intercept.

Hence write the vector equation of the plane passing through the points (2 , 3 , -1) and parallel to the plane obtained above.

[OR]

Show that the line of intersection of the plane $x + 2y + 3z = 8$ and $2x + 3y + 4z = 11$ is co planar with

the line $\dfrac{x+1}{1} = \dfrac{y+1}{2} = \dfrac{z+1}{3}$. Also find the equation of the plane containing them.

28. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

29. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup.