## Class: X <br> Date: 15/01/2019 <br> General Instructions:

Maxi. Marks: $\mathbf{8 0}$
a. All questions are compulsory.
b. The question paper consists of 30 questions divided into four sections $A, B, C$ and $D$.
c. Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section $C$ contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
d. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculator is not permitted.

## SECTION A

## Question numbers 1 to $\mathbf{6}$ carry 1 mark each.

1. If $\sin \alpha=\frac{1}{2}$ and $\cos \beta=\frac{1}{2}$, then find $\alpha+\beta$.

## OR

Find the value of $\frac{\sin 41^{\circ}}{2 \cos 49^{\circ}}-\frac{\sec 32^{\circ}}{\operatorname{cosec} 58^{\circ}}$
2. If $P(2, a)$ is the mid- point of the line segment joining the points $A(6,-5)$ and $B(-2,11)$, find the value of a.
3. If two positive numbers $a$ and $b$ are written as $a x^{3} y^{2}$ and $b=x y^{3}, x, y$ are prime numbers, then find $\operatorname{HCF}(a, b)$.
4. In $\triangle A B C, D E \| B C$. If $D E=\frac{2}{3} B C$ and area of $\triangle A B C=81 \mathrm{~cm}^{2}$, find the area of $\triangle A D E$.
5. For what value of $k$ will $k+9,2 k-1$ and $2 k+7$ are the consecutive terms of an A.P.?
6. For what value of $k$ does the equation $9 x^{2}+3 k x+4=0$ have equal roots?

## OR

State the condition that the equation $a x^{2}+b x+c=0$ has no real roots.

## SECTION B

## Question numbers $\mathbf{7}$ to $\mathbf{1 2}$ carry $\mathbf{2}$ marks each.

7. Find the 10th term from the end of the A.P. 4, 9, 14, 254.

## OR

Find the sum of $1+6+11+16+$ $\qquad$ +81 .
8. Find the value of $p$ if the points $(p, 3),(6,-2)$ and $(-3,4)$ are collinear.
9. Given that $\sqrt{5}$ is irrational number, prove that $3+2 \sqrt{5}$ is an irrational number.

OR
Explain why $11 \times 13 \times 15 \times 17+17$ is a composite number.
10. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Deepak wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Deepak will lose the game.
11. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is:
(i) a black King (ii) a card of red colour.
12. Solve for x and $\mathrm{y}: \quad \mathrm{x}+\frac{6}{y}=6,3 \mathrm{x}-\frac{8}{y}=-8$

## SECTION C

## Question numbers $\mathbf{1 3}$ to $\mathbf{2 2}$ carry 3 marks each.

13. Use Euclid's division lemma to show that cube of any positive integer is either of the form 9 m , $9 m+1$, or $9 m+8$.
14. The mid - point of the line segment joining $(2,3)$ and $(3,-2)$ lies on the line $2 x-y+k=0$. Find the value of $k$.

## OR

If the point $P(x, y)$ is equidistant from the points $A(a+b, b-a)$ and $B(a-b, a+b)$. Prove that $b x=a y$.
15. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm , a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

## OR

A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of $2 \mathrm{~km} / \mathrm{h}$, in how much time will the tank be filled ?
16. If the zeroes of the polynomial $x^{3}-3 x^{2}+x+1$ are $a-b$, $a$ and $a+b$, find the values of $a$ and $b$.
17. If all sides of a parallelogram touch a circle, then prove that it is a rhombus.
18. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

## OR

In the figure, $P A, Q B$ and $R C$ are perpendiculars to $A C . P A=x, Q B=z, R C=y, A B=a$ and $B C=b$. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$

19. $A B$ and $C D$ are two diameters of a circle (with centre $O$ ) perpendicular to each other and $O D$ is the diameter of the smaller circle. If $O A=7 \mathrm{~cm}$, find the area of the shaded region.

20. Find the mode of the marks obtained by 80 students in a class test in mathematics as given below:

| Marks | less than <br> 10 | Less <br> than 20 | Less <br> than 30 | Less <br> than 40 | Less <br> than 50 | Less <br> than 60 | Less <br> than 70 | Less <br> than 80 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No. of | 3 | 8 | 24 | 36 | 49 | 69 | 75 | 80 |

21. If $\cos \theta+\sin \theta=\sqrt{2} \cos \theta$, then show that $\cos \theta-\sin \theta=\sqrt{2} \sin \theta$

OR

If $\cos (A+B)=\frac{1}{2}=\sin (A-B)$. Find $A$ and $B$.
22. The ratio of incomes of two persons is $9: 7$ and the ratio of their expenditures is $4: 3$. If each of them manages to save Rs 2000 per month, find their monthly incomes.

## SECTION D

## Question numbers $\mathbf{2 3}$ to $\mathbf{3 0}$ carry $\mathbf{4}$ marks each.

23. Prove that in a right angle triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
24. If $\tan A+\sin A=m$ and $\tan A-\sin A=n$. Show that $m^{2}-n^{2}=4 \sqrt{m n}$.
25. Draw a triangle ABC with side $\mathrm{BC}=7 \mathrm{~cm}, \angle B=45^{\circ}$, and $\angle \mathrm{A}=105^{\circ}$. Then construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of triangle $\triangle A B C$.
26. The mean of the following distribution is 54 . The frequencies $f_{1}$ and $f_{2}$ in class $20-40$ and $60-80$ are missing. Find the missing frequencies.

| Class | $0-20$ | $20-40$ | $40-60$ | $60-80$ | $80-100$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :--- |
| Frequency | 7 | $\mathrm{f}_{1}$ | 10 | $\mathrm{f}_{2}$ | 13 | 50 |
| OR |  |  |  |  |  |  |

The following distribution gives the daily wages of 60 workers of a factory:

| Daily wages | $20-40$ | $40-60$ | $60-80$ | $80-100$ | $100-120$ | $120-140$ | $140-160$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :--- | :--- |
| Number of workers | 4 | 6 | 10 | 16 | 12 | 8 | 4 |

Convert the above distribution into a less than type cumulative frequency distribution. Draw its ogive and find the median.
27. The present age of a father is equal to the square of the present age of his son. One year ago, the age of the father was 8 times the age of his son. Find their present ages.

OR
A person on tour has Rs 360 for his daily expenses. If he extends his tour for four days, he has to cut down his daily expenses by Rs 3 . Find the original duration of the tour.
28. The height of a cone is 30 cm . A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?
29. The angle of elevation of the top of a tower from two points $P$ and $Q$ at a distance of 4 m and 9 m respectively from the base of the tower and in the same straight line with it are $60^{\circ}$ and $30^{\circ}$. Prove that the height of the tower is 6 m .

OR
An aeroplane at an altitude of 200 m observes the angles of depression of two opposite points on two banks of the river to be $45^{\circ}$ and $60^{\circ}$. Find the width of the river. (Use $\sqrt{3}=1.732$ ).
30. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class in which they are studying, e.g. a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are 4 sections of each class. Find the total number of trees planted by the students of the school.

