INDIAN SCHOOL SOHAR
PRE BOARD EXAMINATION (2018 -19)
MATHEMATICS

CLASS: XII **MAX. MARKS: 100**
DATE: 10/01/2019 **DURATION: 3 HRS**

General Instructions:

a. All questions are compulsory.
b. This question paper contains 29 questions.
c. Questions 1 – 4 in Section A are very short-answer type questions carrying 1 mark each.
d. Questions 5 – 12 in Section B are short-answer type questions carrying 2 marks each.
e. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 marks each.
f. Questions 24 – 29 in Section D are long-answer II type questions carrying 6 marks each.

SECTION A

1. If A is matrix of order m × n and B is a matrix such that AB′ and B′A are both defined, then find the order of matrix B

2. If f (x) = |cos x – sinx|, find f' $\left(\frac{\pi}{3}\right)$

3. What is the integrating factor of the differential equation x log x $\left(\frac{dy}{dx}\right)$ + y = 2 log x ?

4. A plane meets the co-ordinates axis in A, B, C such that the centroid of the triangle ABC is the point (α, β, χ) then find the value of $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\chi}$

[OR]

P is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If *x* co-ordinate of P is 5, find the y co ordinate

SECTION B

5. If f = {(5, 2), (6, 3)}, g = {(2, 5), (3, 6)}, write f o g.

6. Given A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, If $A^{-1} = A'$ find the value of α

7. Find $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx$

8. Find $\int \sqrt{1+\sin 2x}\;dx$

[OR]

Evaluate $\int \frac{x}{x^4+1}\;dx$

9. Find the order and degree of the following differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$

[OR]

Find the general solution of the equation $\frac{dy}{dx}=e^{x-y}$

10. If $\vec{a},\vec{b},$ are unit vectors , find the angle between $\vec{a}$ *and* $\vec{b}$ such that $\sqrt{3}\vec{a}-\vec{b}$ is a unit vector

11. Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six.

12. A die is thrown 5 times. Find the probability that an odd number will come up exactly three times.

SECTION C

13. Let f, g: R → R be two functions defined as f (x) = x + $|x|$ and g (x) = x – $|x|$ ∀ x ∈ R. Then, find f o g and g o f.

14. Find the greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$

[OR]

Show that $2\tan^{-1}\left\{\tan\frac{\alpha}{2}.\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\} = \tan^{-1}\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}$

15. Using the properties of determinant prove that $\begin{vmatrix} a & b-c & b+c \\ c+a & b & c-a \\ a-b & a+b & c \end{vmatrix}$ is divisible by (a+ b + c) and find the quotient.

16. Determine the values of a and b if f(x) is continuous in the interval [0 , 8]

$$f(x)=\begin{cases} a\tan^{-1}\dfrac{1}{x-4} & 0\le x<4 \\ \dfrac{\pi}{2} & x=4 \\ b\tan^{-1}\dfrac{2}{x-4} & 4<x<6 \\ \sin^{-1}(7-x)+a\dfrac{\pi}{4} & 6\le x\le 8 \end{cases}$$

[OR]

Find the values of a and b so that the function $f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x} & x<\dfrac{\pi}{2}\\ a & x=\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & x>\dfrac{\pi}{2}\end{cases}$

is continuous at x = $\frac{\pi}{2}$

17. If y = $\tan^{-1}\left(\frac{1}{x^2+x+1}\right)+\tan^{-1}\left(\frac{1}{x^2+3x+3}\right)+\tan^{-1}\left(\frac{1}{x^2+5x+7}\right)$+…..+ n terms, then prove that $\frac{dy}{dx}=\frac{1}{(x+n)^2+1}-\frac{1}{x^2+1}$

18. Show that the equation of normal at any point on the curve x = 3cos θ – $\cos^3\theta$, y = 3sinθ – $\sin^3\theta$ is 4 (y $\cos^3\theta$ – x $\sin^3\theta$) = 3 sin 4θ.

19. Evaluate $\int 3x+5\sqrt{2x^2+3x+7}\;dx$

20. Evaluate $\int_0^1 x\left(\tan^{-1}x\right)^2 dx$

21. Find the general solution of (1 + tany) (dx – dy) + 2xdy = 0.

[OR]

Find the general solution of $y^2\,dx + (x^2 - xy + y^2)\,dy = 0$.

22. Show that the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are co planar ,then $\vec{a}+\vec{b}$, $\vec{c}+\vec{b}$ and $\vec{a}+\vec{c}$ are co planar.

23. If a variable line in two adjacent positions has direction cosines *l*, *m*, *n* and $l+\delta l, m+\delta m, n+\delta n$, show that the small angle δθ between the two positions is given by $\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$

SECTION D

24. If A = $\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, then find A^{-1} and hence solve the system of equations x - 2y = 10 ,

2x – y - z = 8 , - 2y + z = 7

[OR]

Given $A = \begin{bmatrix} 2 & 2 & 4 \\ 4 & 2 & 4 \\ 2 & 1 & 5 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ find BA and solve the system of equations

$y + 2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$

25. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$

26. Find the area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$.

[OR]

Find the area of the region above the *x*-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.

27. Find the coordinates of the point where the line through (3, – 4, – 5) and (2, –3, 1) crosses the plane passing through three points (2, 2, 1), (3, 0, 1) and (4, –1, 0)

[OR]

Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

28. A company makes 3 models of calculators: A, B and C at factory I and factory II. The company has orders for at least 6400 calculators of model A, 4000 calculator of model B and 4800 calculator of model C. At factory I, 50 calculators of model A, 50 of model B and 30 of model C are made every day; at factory II, 40 calculators of model A, 20 of model B and 40 of model C are made every day. It costs Rs 12000 and Rs 15000 each day to operate factory I and II, respectively. Find the number of days each factory should operate to minimise the operating costs and still meet the demand.
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29. A discrete random variable X has the following probability distribution

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2 + C$

Find the mean and variance of the distribution.

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