## INDIAN SCHOOL SOHAR

## FINAL EXAMINATION (2018-19)

MATHEMATICS
STD: XI
MAX. MARKS: 100
DATE: 28.02.19

## General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1-4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying $\mathbf{2}$ marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

## SECTION - A

## Questions 1 to 4 carry 1 mark each

1. Check validity of the following statement:

P: 125 is divisible by 5 and 8
2. If the line joining two points $\mathrm{O}(0,0)$ and $\mathrm{B}(\sqrt{3}, 1)$ is rotated about O in anti-clock wise direction through an angle of $15^{\circ}$. Find the equation of the line in new position.

## OR

Find the equation of the line passing through the intersection of the lines $x+y+3=0$ and $x-y+2=0$ and having $y$-intercept equal to 4 .
3. Evaluate: $\lim _{x \rightarrow 1} \frac{1-x^{\frac{1}{3}}}{1-x^{\frac{2}{3}}}$
4. Given that $N=\{1,2,3, \ldots, 20\}$, then write the subset $A$ of $N$, whose element are represented by $x+10$, where $x \in N$.

## SECTION - B

## Questions 5 to $\mathbf{1 2}$ carry 2 marks each.

5. Redefine the function which is given by $\mathrm{f}(\mathrm{x})=|x-1|+|1+x|,-2 \leq \mathrm{x} \leq 2$
6. Find the radius of the circle in which a central angle of $60^{\circ}$ intercepts an arc of length 37.4 cm .
7. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?
8. The first term of a G.P exceeds the second term by 2 and the sum to infinity is 50 . Find the G.P

## OR

The difference between any two consecutive interior angles of a polygon is $5^{\circ}$. If the smallest angle is $120^{\circ}$, find the number of the sides of the polygon.
9. Find the ratio in which the line segment joining the points $(4,8,10)$ and $(6,10,-8)$ is divided by the XY-plane.
10. Evaluate the following limit: $\lim _{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x-\cos x}{x-\frac{\pi}{6}}$

OR
Let $f(x)=\left\{\begin{array}{c}x+2, x \leq-1 \\ c x^{2}, x>-1\end{array}\right.$ find c if $\lim _{x \rightarrow-1} f(x)$ exists.
11. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75 , what is the probability of passing the Hindi examination?

## OR

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.
12. Check the validity of the following statement by using contrapositive method. If $x, y \in Z$ are such that $x$ and $y$ are odd, then $x y$ is odd.

## SECTION - C

## Questions 13 to 23 carry 4 marks each.

13. Find the derivative of $\cos (3 x+1)$ using the first principle.

## OR

Find $\frac{d y}{d x}$ when $\mathrm{y}=\frac{(3 x+1) \sin ^{2} x}{(x-1)}$
14. Find the domain and range of the function $\mathrm{f}(\mathrm{x})=\sqrt{25-x^{2}}$
15. Prove that: $\cos ^{4} \frac{\pi}{8}+\cos ^{4} \frac{3 \pi}{8}+\cos ^{4} \frac{5 \pi}{8}+\cos ^{4} \frac{7 \pi}{8}=\frac{3}{2}$
16. Find the general solution : $\sin \theta+\sin 3 \theta+\sin 5 \theta=0$
17. Prove the following by using the principle of mathematical induction for all $n \in N$ :
$11^{n+2}+12^{2 n+1}$ is divisible by 133
18. Find the square root of the following: $-5+12 \mathrm{i}$
19. How many numbers greater than 1000000 can be formed by using the digits $1,2,0,2,4$, 2,4 ?

## OR

The letters of the word 'GREEN' are written in all possible orders and these words are written out as in a dictionary, what will be the $32^{\text {nd }}$ word? Find the rank of the word 'GREEN'
20. Find the sum of the following series up to $n$ terms: $3+7+13+21+31+$
21. If $A$ and $G$ be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$
22. Show that the locus of the mid-point of the distance between the axes of the variable line $\mathrm{x} \cos \alpha+\mathrm{y} \sin \alpha=\mathrm{p}$ is $\frac{1}{x^{2}}+\frac{1}{y^{2}}=\frac{4}{p^{2}}$ where p is a constant.

## OR

If one diagonal of a square is along the line $8 x-15 y=0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.
23. In a class of 60 students, 30 opted for Physics, 32 opted for Chemistry and 24 opted for both Physics and Chemistry. If one of these students is selected at random, find the probability that (i) The student opted for Physics or Chemistry. (ii) The student has opted neither Physics nor Chemistry. (iii) The student has opted Chemistry but not physics.

## SECTION - D

## Questions 24 to 29 carry 6 marks each.

24. Out of 100 students, 15 passed in Physics, 12 passed in Mathematics, 8 in Chemistry, 6 in Physics and Mathematics, 7 in Mathematics and Chemistry, 4 in Physics and Chemistry, 4 in all the three. Find how many passed in i) in Mathematics only ii) in more than one subject iii) exactly in 2 subjects.
25. Show that $\sin ^{3} x+\sin ^{3}\left(\frac{2 \pi}{3}+x\right)+\sin ^{3}\left(\frac{4 \pi}{3}+x\right)=-\frac{3}{4} \sin 3 \mathrm{x}$

## OR

If $\mathrm{x} \cos \theta=y \cos \left(\theta+\frac{2 \pi}{3}\right)=z \cos \left(\theta+\frac{4 \pi}{3}\right)$, then find the value of $\mathrm{xy}+\mathrm{yz}+\mathrm{zx}$.
26. Calculate mean and Standard Deviation for the following distribution.

| Classes | $0-30$ | $30-60$ | $60-90$ | $90-120$ | $120-150$ | $150-180$ | $180-210$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 9 | 17 | 43 | 82 | 81 | 44 | 24 |

27. Solve the following system of inequalities graphically:

$$
x-2 y \leq-1,2 x+y \geq 3, y<3, x, y \geq 0
$$

28. Find $\mathrm{a}, \mathrm{b}$ and n in the expansion of $(\mathrm{a}+\mathrm{b})^{\mathrm{n}}$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.

## OR

The sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^{2}}\right)^{m}, \mathrm{x} \neq 0$, $m$ being a natural number, is 559 . Find the term of the expansion containing $\mathrm{x}^{3}$.
29. Derive the standard equation of the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$. Also find the coordinate of the foci, the eccentricity and the length of latus rectum of the conic: $x^{2}+16 y^{2}=16$

## OR

Find the equation of the circle passing through the points $(2,3)$ and $(-1,1)$ and whose centre is on the line $\mathrm{x}-3 \mathrm{y}-11=0$.

