INDIAN SCHOOL SOHAR
TERM I EXAMINATION (2019-20)
MATHEMATICS

CLASS: XII | **MAX. MARKS: 70**
DATE: 04/11/2019 | **DURATION: 3 HOURS**

General Instructions:

a) All the questions are compulsory.
b) The question paper consists of 36 questions divided into 4 sections A, B,C, and D.
c) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each.
d) Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
e) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
f) Use of calculators is not permitted.

	SECTION A			
1.	A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ is equal to			
	a) 2AB	b) 2BA	c) A + B	d) AB
2.	If A = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then A^{100} is equal to			
	a) 2^{100} A	b) 2^{99} A	c) 100 A	d) 299 A
3.	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non co planar vectors and λ is a real value then $\left[\lambda\left(\vec{a}+\vec{b}\right)\ \lambda^2\vec{b}\ \lambda\vec{c}\right] = \left[\vec{a}\ \left(\vec{b}+\vec{c}\right)\ \vec{b}\right]$ is possible for			
	a) No value of λ	b) one value of λ	c) Two values of λ	d) Three values of λ
4.	In a college, 30% students fail in physics, 25% fail in mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in physics if she has failed in mathematics is			
	a) $\frac{1}{10}$	b) $\frac{2}{5}$	c) $\frac{9}{20}$	d) $\frac{1}{3}$
5.	The point which does not lie in the half plane $2x + 3y - 12 \le 0$ is			
	a) (1 , 2)	B) (2 , 1)	C) (2 , 3)	D) (-3 , 2)

6.	The principal value branch of $\sec^{-1}$ is			
	a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$	c) $(0, \pi)$	d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
7.	Let A and B be two events such that P (A) = 0.6, P (B) = 0.2, and P (A | B) = 0.5. Then P (A' | B') equals			
	a) $\frac{1}{10}$	b) $\frac{3}{10}$	c) $\frac{3}{8}$	d) $\frac{6}{7}$
8.	If $\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = ax + b \log\left\| 4e^x + 5e^{-x} \right\| + C$ then			
	a) $a = \frac{-1}{8}, b = \frac{7}{8}$	b) $a = \frac{-1}{8}, b = \frac{-7}{8}$	c) $a = \frac{1}{8}, b = \frac{7}{8}$	d) $a = \frac{1}{8}, b = \frac{-7}{8}$
9.	The lines x = ay + b , z = cy + d and x = a'y + b , z = c'y + d are perpendicular to each other if			
	a) $\frac{a}{a'} + \frac{c}{c'} = 1$	b) a a' + c c' = 1	c) $\frac{a}{a'} + \frac{c}{c'} = -1$	d) a a' + c c' = -1
10.	The length of the perpendicular drawn from the point P(3, – 4, 5) on the z-axis. is			
	a) 3	b) 4	c) 5	d) $5\sqrt{2}$
11.	$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then the value of λ is ___________________ **OR** If $\vec{a} = -2\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = a\hat{i} - k$ are collinear , then the value of k is ________________			
12.	The sum of the products of elements of any row with the co-factors of corresponding elements of another row is equal to __________.			
13.	The number of points at which the function f (x) = $\frac{1}{\log\|x\|}$ is discontinuous is -____________			
14.	The values of a for which y = x^2 + ax + 25 touches the axis of x are_______. OR The rate of change of volume of a sphere with respect to its surface area, when the radius is 2 cm, is_______.			
15.	Let f : R → R be defined by f (x) = $3x^2 - 5$ and g : R → R by g (x) = $\frac{x}{x^2+1}$ then $g \circ f(x)$ *is* _________________			

16.	Find the value of $\begin{vmatrix} \operatorname{cosec}^2\theta & \cot^2\theta & 1 \\ \cot^2\theta & \operatorname{cosec}^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix}$
17.	If $\int_0^1 \frac{e^t}{1+t}\,dt = a$, then what is the value of $\int_0^1 \frac{e^t}{(1+t)^2}\,dt$?
18.	Evaluate $\int \frac{dx}{\cos^2 x\,(1-\tan x)^2}$
19.	Evaluate $\int_{-\pi}^{\pi} (1-x^2)\sin x \cos^2 x\,dx$
20.	Write the differential equation representing the family of curves y = e^{2x} (a + bx), where 'a' and 'b' are arbitrary constants.
SECTION B	
21.	Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive and symmetric OR Simplify $\tan^{-1}\left[\frac{1+\tan x}{1-\tan x}\right]$.
22.	Let f be continuous on [a, b] and differentiable on the open interval (a,b). then f is increasing in [a,b] if f '(x) > 0 for each x ∈ (a, b) - Prove by mean value theorem
23.	A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.
24.	If $\vec{p} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{q} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude $5\sqrt{3}$ units perpendicular to the vector q and coplanar with vectors $\vec{p}$ and $\vec{q}$. OR If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$
25.	Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.
26.	In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?
SECTION C	
27.	Let A = R – {2} and B = R – {1}. If f : A $\to$ B is a function defined by f(x) = $\frac{x-1}{x-2}$ is f is one-one and onto? Is f is invertible ? If yes , find f^{-1}.
28.	Differentiate $\tan^{-1}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$ with respect to $\cos^{-1} x^2$

	OR If $x^m y^n = (x + y)^{m+n}$, Prove that $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$
29.	Show that the differential equation $2y\, e^{\frac{x}{y}}\, dx + \left(y - e^{\frac{x}{y}} \right) dy = 0$ Is homogeneous and find its particular solution, given that, x = 0 when y = 1.
30.	Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)^2}$
31.	A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATA NAGAR. Bag I contains 4 red and 2 green balls and Bag II contains 3 red and 5 green balls. One ball is transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. The ball so drawn is found to be green in colour. Find the probability that the transferred ball is also green.
32.	A small firm manufactures chairs and tables. Market demand and available resources indicate that the combined production of chairs and tables should not exceed 50 units per day. It takes 30 minutes to manufacture a chair and 1 hour to manufacture a table. A maximum of 40 man-hours per day are available. The profit on each chair is Rs. 40 and profit on each table is Rs. 60. Determine how many each of chairs and tables should be manufactured per day in order to maximize the profit. What is the maximum profit? Formulate LPP and solve graphically.
SECTION D	
33.	Solve the system of equations using matrices Y + 2z = 7 , x – y = 3 , 2x + 3y + 4z = 17 OR If a + b + c $\neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = 0 , then prove that a = b = c
34.	Using integration find the area of the triangle formed by positive x axis and tangents and normal to the circle $x^2 + y^2 = 4$ at (1 , $\sqrt{3}$)
35.	A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi-circular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of semi-circular ends is π : π + 2. OR Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.
36.	A line with direction ratios < 2, 2, 1 > intersects the lines, $\frac{x-7}{3} = \frac{y-5}{2} = \frac{z-3}{1}$ and $\frac{x-1}{2} = \frac{y+1}{4} = \frac{z+1}{3}$ at the points P and Q respectively. Find the length and the equation of the intercept PQ.

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