INDIAN SCHOOL SOHAR
UNIT TEST (2019 -2020)
MATHEMATICS

CLASS: XII **MAX. MARKS: 40**
DATE: 15/05/2019 **DURATION: 2 HRS**

General Instructions

a. *All questions are compulsory.*
b. *The question paper consist of 18 questions divided into four sections A, B,C and D. Section A comprises of 10 questions of one mark each, section B comprises of 3 questions of two marks each and section C comprises of 3 questions of four marks each. And section D comprises of 2 questions of six marks each.*
c. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
d. *There is no overall choice. However, internal choice has been provided in 01 question in each division.*
e. *Use of calculators is not permitted.*

SECTION – A

1. The objective function of the linear programming problem is Z = Px + Qy , the corner points of the feasible region are (0 , 10) , (15 , 15) , (0 , 20) .If Z yields the maximum at (15 , 15) and (0 , 20) , find the relation between P and Q .
2. Write the corner points of the feasible region formed by the constraints $x + y \le 6$, $x + y \ge 3$, $x \ge 0$ and $y \ge 0$
3. What is the relation between the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-3}{-1}$ and the plane π = x – 2y – z ?
4. If a line makes angle α , β *and* γ with the coordinate axes, what is the value of cos2α + cos 2β + cos 2γ .
5. Let $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right|$ = 3 and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$ and cross product of both $\vec{a}$ and $\vec{b}$ is unity .Find the angle between $\vec{a}$ and $\vec{b}$
6. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors , find the value of $\left|\vec{a} + \vec{b} + \vec{c}\right|$
7. Find the value of x , if $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
8. Find the greatest value of $\left(\sin^{-1} x - \frac{\pi}{2}\right)^2 + \frac{\pi^2}{16}$
9. Let $f : R \to R$ is defined as $f(x) = \frac{x}{x^2 + 1}$, find the inverse of f.

10. Apply $R_1 \rightarrow R_1 - 2R_2$ in the following matrix equation

$$\begin{pmatrix} 3 & -7 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 9 & -4 \\ 2 & 6 \end{pmatrix}$$

[OR]

If $A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$, then find the value of $7A - 3A^T$

SECTION C

11. Prove the following $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

[OR]

Solve for 'x' $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$

12. Find the equation of the line through the point (3, 0, 1) and parallel to the planes x+2y = 0 and 3y – z = 0

13. Find the value of α if $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then $A + A^T = I$

SECTION C

14. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs)

FROM /TO	A	B
D	6	4
E	3	2
F	2.5	3

How should the supplies be transported in order that the transportation cost is minimum? Derive the constraints and optimal function.

[OR]

If a young man rides his motor-cycle at 25 km per hour, he had to spend Rs 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to Rs

5 per km and rate of pollution also increases. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour?. Express this problem as an L.P.P.

15. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$

16. Let f : N →R be a function defined as f (x) = $4x^2 + 12x + 15$. Show that f : N→ S, where, S is the range of f, is invertible. Find the inverse of f.

SECTION D

17. Find the inverse of $\begin{pmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ by using elementary transformation

[OR]

Let A = $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, show that $(aI + bA)^n = a^n I + n a^{n-1} b A$

18. Prove that the lines $\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z}{2}$ and $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{-1}$ are co planar. Find the equation of the plane contains theses lines.

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