INDIAN SCHOOL SOHAR No. of pages: 6 + 1 graph

TERM I EXAMINATION (2022-23)

MATHEMATICS (CODE -041)

CLASS: XII **MAX. MARKS: 80**

DATE: 26/9/22 **TIME: 3 Hrs.**

General Instructions:

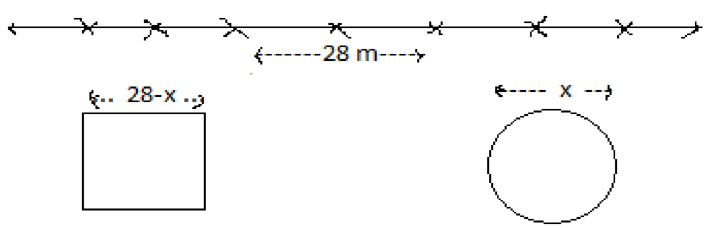
1. This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 20 Multiple Choice Questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 Case based questions of 4 marks each with sub parts.

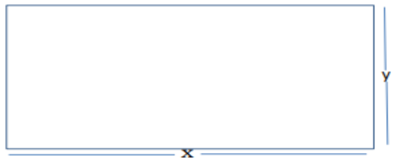
	SECTION – A **(This Section comprises of 20 multiple choice questions of 1 mark each)**	
1.	If a relation R in R is defined as **a R b** if a$\geq b$. Then R is a) an equivalence relation b) reflexive, transitive but not symmetric c) symmetric, transitive but not reflexive d) neither transitive nor reflexive but symmetric	**MARKS** 1
2.	Let N be the set of natural numbers and the function f : N → N be defined by f (n) = 5n - 3 ∀ n ∈ N. Then f is a) surjective b) one-one and onto c) injective d) f is not defined	1
3.	The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is equal to a) $\frac{\pi}{2}$ b) $\frac{3\pi}{2}$ c) $\frac{5\pi}{2}$ d) $\frac{7\pi}{2}$	1
4.	Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$. a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{-\pi}{3}$	1
5.	If A and B are matrices of same order, then (AB'–BA') is a a) skew symmetric matrix b) null matrix c) symmetric matrix d) unit matrix	1
6.	If $\begin{vmatrix} 2x & 5 \\ 6 & x \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 10 & 5 \end{vmatrix}$ then x equal to a) $\sqrt{5}$ b) 5 c) $\frac{1}{5}$ d)± 5	1

7.	Differentiation of x^x with respect to ($x \log x$ +1) is a) 1 + log x b) x log x + x^x c) x log x +1 d) x^x	1
8.	If x = at^2 and y = 2at then $\frac{d^2y}{dx^2}$ is a)$\frac{-1}{2a\,t^3}$ b) -2at^3 (c) $\frac{-1}{t^2}$ d) t^2	1
9	Evaluate $\int \frac{1}{sin^2x\ cos^2x}dx$ a) -tanx + cotx + c b) cotx + tanx +c c) tan^3x + cotx + c d) tanx – cotx + c	1
10	The interval in which the function f(x)= $2x^3$ -$15x^2$ +36x +1 is decreasing is: a)$(3, \infty)$ $b)(-\infty, 2)$ c) (2,3) d) $(-\infty, 2)\mathrm{U}(3, \infty)$	1
11	Let A = {a, b, c} and the relation R be defined on A as follows: R = {(a, a), (b, c), (a, b)}. Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive. a) $\{(b,b),(c,c),(a,c)\}$ b) $\{(b,b),(a,c)\}$ c) $\{\ (a,c)\}$ d) $(c,c),(a,c)\}$	1
12	Evaluate : $\tan^{-1}$(tan $\frac{5\pi}{6}$) a) $\frac{\pi}{6}$ b) $\frac{-\pi}{6}$ c) $\frac{-\pi}{3}$ d) $\frac{\pi}{3}$	1
13	The values of a, b, c, and d from the following are: $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$ a)a=2,b=1,c=3,d=4 b) a=1, b=-2, c= 3, d= 4 c) a= 1,b=2, c= 4, d= 3 d) a=1, b=2, c=3, d=4	1
14	If $x^y = e^{x-y}$ then $\frac{dy}{dx}$ is equal to a) $\frac{\log x}{1+logx}$ b) $\frac{\log x}{(1-logx)^2}$ c) $\frac{\log x}{(1+logx)^2}$ d) $\frac{x}{(1+logx)^2}$	1
15	If y = a sin px + b cos px, then $\frac{d^2y}{dx^2}$ is equal to a) -p^2y b) p y c) p^2y d) -p y	1

16	The length x of a rectangle is decreasing at the rate of 5 cm/sec and the width y is increasing at the rate of 4 cm/sec. When x = 8cm and y = 6cm, find the rate of change of the perimeter. a) -3 cm/sec b) -2 cm/sec c)14cm/sec d) 9 cm/sec	1
17	$\int xe^{(1+x^2)}dx$ is equal to a) $\frac{e^{(1+x^2)}}{2}$ + c b) $e^{(1+x^2)} + c$ c)$xe^{(1+x^2)} + c$ d) $\frac{(1+x^2)}{2}$ + c	1
18	$\int e^x(1 - cotx + cosec^2x)dx$ is equal to a) e^x (cotx) +c b) e^x ($cosec^2x$) +c c) e^x ($-cosec^2x$) +c d) e^x (1 - cotx) +c	1
19	Evaluate: $\tan^{-1}\sqrt{3} - sec^{-1}(-2) + cosec^{-1}\left(\frac{2}{\sqrt{3}}\right) + cos^{-1}(\frac{-1}{\sqrt{2}})$ a) $\frac{3\pi}{2}$ b) $\frac{3\pi}{4}$ c)0 d) $\frac{\pi}{4}$	1
20	Corner points of the feasible region determined by the system of linear constraints are (0, 2), (1, 1) and (2, 0). Let Z = px + qy ($p > 0, q > 0$), then the condition on p and q so that minimum z occurs at (2,0) and (1,1) is: a)p=2q b) p= $\frac{q}{2}$ c) p=3q d) p=q	1
	SECTION B **(This section comprises of very short answer type questions (VSA) of 2 marks each)**	
21	Find: $\int sin^{-1}(2x)dx$	2
22	Find: $\int \frac{\cos 2x}{(sinx+cosx)^2}\, dx$	2
23	Let $f: N \to N$ be defined by f(n) = $\begin{cases} \frac{n+1}{2}, n \text{ is odd} \\ \frac{n}{2}, n \text{ is even} \end{cases}$ Check the injectivity and surjectivity of the function. **OR** Check whether the relation R in R defined by R = {(a, b): a ≤ b^3} is reflexive, symmetric or transitive.	2
24	If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}.	2

25	Find the value of the constant k so that the function given below is continuous at x= 0. F(x) = $\begin{cases} \frac{1-\cos 2x}{4x^2}, x \neq 0 \\ k,\ x = 0 \end{cases}$ **OR** If y = 3 cos (log x) + 4 sin(log x) , then show that: $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y$ =0	2
SECTION C **(This section comprises of short answer type questions (SA) of 3 marks each)**		
26	Differentiate the following function w.r.t. x : y =$(sinx)^x + sin^{-1}\sqrt{x}$ **OR** If y = $e^{x\,sin^2x}$+$(sinx)^x$, find $\frac{dy}{dx}$	3
27	Find the area of the ellipse x^2 + 9 y^2 = 36 using integration. **OR** Find the area of the region bounded by the curve y^2 = 8x and the line x = 2.	3
28	The area between x = y^2 and x = 4 is divided into two equal parts by the line x = a, find the value of a.	3
29	Find the equation of the line joining A(1, 3) and B (0, 0) using determinants and find k if D(k, 0) is a point such that area of triangle ABD is 3sq units. **OR** Using cofactors of the elements of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ Evaluate: $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33}$	3
30	Find $\int \frac{x^3}{x^4+3x^2+2}\ dx$	3
31	Solve the following linear programming problem graphically. Maximize Z = x+ 2y Subject to constraints; x + 2y ≥ 100 2x – y ≤ 0 2x + y ≤ 200 x,y ≥ 0	3

	SECTION D **(This section comprises of long answer type questions (LA) of 5 marks each)**	
32	Let Z be the set of all integers and R be the relation on Z defined as R = {(a, b): a, b ∈ Z, and (a - b) is divisible by 5.} Prove that R is an equivalence relation.	5
33	Solve the following system of equations by matrix method, where x$\neq 0, y \neq 0,$ $z \neq 0$ $\frac{2}{x} - \frac{3}{y} + \frac{3}{z}$ =10 $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ =10 $\frac{3}{x} - \frac{1}{y} + \frac{2}{z}$ =13	5
34	Find: $\int \frac{sinx}{sin^3x+cos^3x}\,dx$ **OR** Find: $\int \frac{1}{cos^4x+sin^4x}\,dx$	5
35	Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. **OR** A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.	5
	SECTION E **(This section comprises of case study based questions of 4 marks each)**	
36	Geeta bought a wire of length 28 m which is to be cut in to two pieces. One of the pieces is to be made into a square and the other into a circle as shown in the figure.  	
	i) Find the area of the circle in terms of x	1
	ii) Find the area of the square in terms of x.	1
	iii) Find the length of the wire bent into the form of square. **OR** iii) Find the length of the wire bent into the form of circle.	2

37	Mohan wants to donate a rectangular plot of land for a hospital in his village. When he was asked to give dimensions of the plot, he told that if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300 m^2 , but if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same.  Answer the following questions:	
	i) Write the equation in terms of X and Y using matrix equation.	1
	ii) Find the value of x.	1
	iii) Find the area of the rectangular field. **OR** iii)Find the perimeter of the rectangular field.	2
38	P(x) = $-3x^2 + 84x + 1500$ is the total profit function of a company, where x is the production of the company. 	
	i) What will be the production when the profit is maximum? Also find the maximum profit.	2
	ii) Check in which interval the profit is : a)strictly decreasing b) strictly increasing	2

----------------------THE END ------------------